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- [x] Final Invoice or Copy of Last Invoice Serving as Final
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TECHNOLOGY ASSESSMENT FOR STRUCTURAL ANALYSIS OF HOT, HIGHLY-LOADED STRUCTURES

SUBMITTED TO

MCDONNELL DOUGLAS ASTRONAUTICS COMPANY

A TECHNICAL REPORT

ENTITLED

TECHNOLOGY ASSESSMENT FOR STRUCTURAL ANALYSIS OF HOT, HIGHLY-LOADED STRUCTURES

SUBMITTED TO

MCDONNELL DOUGLAS ASTRONAUTICS COMPANY

BY

G.J. SIMITSES[1] AND R. RIFF[2]
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1. INTRODUCTION

Design of future hypersonic vehicles will require analytical capabilities for accurately predicting local and global response of the system, when subjected to transient, cyclic or static thermomechanical loads with large excursion. Some of these needed analytical tools are being currently developed, while others are not yet in existence.

These future vehicles are in the feasibility study stage and only gross mission requirement have been set with the purpose of identifying technical needs. Some of these vehicles include the National Aerospace Plane (NASP), hypersonic airbreathing missiles, and military hypersonic aircraft such as the Advanced Tactical Fighter (ATF). The common features of such vehicles is that they must operate in altitudes from sea level to the fringes of outer space and at extremely high speeds. This, of course, suggests large thermomechanical loads of short and long duration.

The technical needs, some of which are currently addressed, include a highly integrated airframe; revolutionary multi-component propulsion system; control systems that use advanced computers and adaptive intelligence; high-strength, high-temperature fully reusable materials; and design codes based on aerodynamic, propulsion and structural analyses that will yield reliable results quickly (use of supercomputers is suggested here).

The emphasis, in this report, is placed on structure-related problems. First, the needed analytical capabilities for predicting the response of such structural configurations to large thermomechanical loads, are identified. Next, a state-of-the-art presentation is made of the theoretical formulations and of all non-secret (found in the open literature and in commercially available packages) analytical capabilities for various structural configurations subjected to high temperature environments. Finally, needed research and development studies are discussed, which will close the gap between the currently available and the needed analytical capabilities.

Before closing, additional tasks will be discussed, which have some bearing on the design and manufacture of such vehicles.

2. STATEMENT OF THE NEEDS

2.1 Aerothermoelasticity

Accurate prediction of the structural response of future hypersonic vehicles strongly depends on the success with which the operating environment of these vehicles, especially the aerothermomechanical loads, can be simulated and calculated. Components of the structure are expected to operate in a severe environment that includes high-temperature and high-velocity flows. Such flows represent a source of energy that can induce and sustain large-amplitude vibratory stresses, high temperatures, and possibly result in fluid induced structural instabilities. The most severe and damaging stresses and strains are those caused by the steep thermal gradients and rates. These transient stresses and strains are also the most difficult to predict, primarily because the temperature distribution gradients and rates are not well known or readily predictable. Progress, therefore, must be made towards developing a general method for predicting unsteady heat transfer on the structure.

The design of hot-section components is generally accomplished by examining the fluid and the solid, separately. Typically, the analysis that is required to support the design process is performed by separate groups using different techniques and computer codes. The prediction of thermally induced deformation is accomplished in a sequence of analyses. First, heating rates are predicted on aerodynamic surfaces by using either empirical methods or finite difference methods. Then the structural temperatures are determined by using a network-type thermal analysis or via a finite element thermal analysis. Finally, structural deformations and stresses are computed by a finite element analysis using temperatures as input data. There are several recognized shortcomings of the overall approach: (1) the sequence of analyses is relatively inefficient because incompatible models are used in the three separate analyses that make data transfer difficult, (2) the approach assumes that the heat transfer between the fluid and the structure and the thermally induced deformations have negligibly small effect on aerodynamic heating, and (3) Since the problem is essentially a coupled one, any separate treatment of each part (fluid and solid) requires the introduction of a complex set of artificially devised boundary conditions at the common interface, to take care of the complex flow and heat transfer phenomena. The solid region is subjected to pressure and temperature resulting in displacements and stresses. The fluid flow is governed by the velocity, pressure and temperature, as well as the boundary displacements.

Yet there exist important design problems where fluid-thermal-structural interactions are important. One example of this is the metallic thermal protection system tested at Mach seven, in the eight-foot high temperature wind tunnel at the NASA Langley Research Center[1]. A second example of problems where flow, thermal, and structural interactions are important is the scramjet engine for the national aerospace plane(NASP).

There are two approaches that can be taken for this category of problems (1) modify/apply/disseminate existing computational fluid dynamics tools in response to current needs, and (2) develop new technology that will enable more accurate computation of the time-averaging and unsteady aerothermodynamic load prediction, providing the needed boundary conditions to improve structural analysis.

In the first approach, the objective is to develop a three-dimensional viscous flow analysis capability, in order to accurately predict the aerothermodynamic loads. This capability should concern both, time averaging (static) methods as well as time-dependent (dynamic) methods. Experimental verification of the theoretical predictions is highly recommended.

In the second approach, the objective is to devise an integrated solution of the system governing equations. These equations include the Navier-Stokes equations, the heat transfer equation, and the equation of motion for the solid. Note that all three sets of equations are coupled. Another possible method of attack in this second approach is to employ a boundary integral representation for the entire system by making simultaneous use of Boundary Element and Finite Element Methods.

2.2 Materials and Their Behavior

The operational environments of hypersonic vehicles demand structural materials that can function under very high temperature conditions and for a prolonged duration of time. The wing skin, for example, must be able to retain its stiffness, strength, dimensional stability and other properties during long flights in a temperature environment of 2500^{o}F or more. The engine components must operate at even higher temperatures. New materials such as fiber reinforced metals and ceramics have the potential for satisfying both the high strength and high temperature requirements. In order to fully utilize the capabilities of these materials, their deformation properties must be analyzed carefully. The structural alloys to be used in the system exhibit complex thermomechanical behavior that is inherently time dependent and hereditary. Hereditary in the sense that current behavior depends not only on current conditions but on thermomechanical history. In addition, this new generation of alloys possesses strong directional characteristics, e.g. directionally solidified metals and metal-matrix composite materials. In high temperature applications these materials exhibit all the complexities of conventional alloys (e.q., creep, relaxation, recovery, rate sensitivity) and, in addition, exhibit further complexities because of their strong anisotropy. Regardless of the status of these materials, their behavior definitely falls in the nonlinear regime, with regard to dependence on temperature, time and kinematics.

The mechanics applicable to these special materials is in an embryonic state of development. The principles of thermoelasticity for isotropic materials have to be extended to composites and other anisotropic materials, and to inelastic state of deformation. Microstructural mechanisms involved in elevated temperature effects have to be examined with regard to creep and fracture phenomena. What is lacking is a basic understanding of the specific mechanisms of thermomechanical deformation and damage that occur at all levels of the material microstructure. Accounting for non-equilibrium thermodynamic processes (under high strain rates and thermomechanical loading condition) in describing material behavior is also needed.

Experimentation to identify the underlying mechanisms of thermomechanical response, for obtaining basic data for the new constitutive laws and for verification of the theories is perhaps the first important research needed in this area. This is a challenge of major proportion because elevated temperature is a hostile environment for laboratory equipment. Nevertheless, effort in this direction must accompany any development of advanced constitutive models for these materials.

2.3 Formulation (Nonlinear Kinematics)

The expected configurations are extensions of current aerospace vehicles. As such, the structural geometry will consist of the integration of shell-like and beam-like structures. The loading on these structures, based on the anticipated mission requirements, will consist of aerothermomechanical loads with large excursions and high dependence on time. Therefore, large deflections and large rotations may form an important consideration. Moreover, large strains and high strain rates may results due to the nature of the loading.

The question of the nonlinearity of the kinematics, with regard to "how large is large?" in small strain formulations is of extreme importance in the derivation of the governing equations. Theoretically, one defines large deformations of shell- like structures, as those that are not constrained by the magnitude of translation and rotation (excluding rigid body motions). Large deformation in small strain formulation, results because of large rotations. A new approach to the derivation of geometrically non-linear "small strain" thin body theories, which provides understanding of their range of applicability and gives a deeper insight into questions of consistency, is given in the works of Pietraszkiewicz [2]-[4]. In these references a polar decomposition theorem is applied to shell-like structures according to which the deformation at each point of the structure may be decomposed exactly, apart from a rigid-body translation, into a pure stretch along the principal directions of strain and a rigid body rotation. This makes it possible to impose restrictions on strains and rotation, independently. As a result it is suggested to classify shell theories which assume small strain of $O(\eta)$, $(\eta \ll 1)$ according to the magnitude of the rotation angle ω of the shell material element as follows:

small rotation:	$\omega = O(\eta)$	, i.e. $\omega \ll 1$;	(1)
moderate rotations:	$\omega = O(\eta^{1/2})$	, i.e. $\omega^2 \ll 1$;	(2)
large rotations:	$\omega = O(\eta^{1/4})$	, i.e. $\omega^4 \ll 1$;	(3)
unrestricted rotations:	$\omega \geqslant O(1)$		(4)

According to this classification it seems that the moderate rotation theory in an updated coordinate system will suffice to calculate the kinematics for most of the nonlinear pre-and post-critical shell-like problems.

The question of applicability of small strain formulations seems to be more difficult. This applicability is affected by strain itself in static problems and by strain rates in dynamic problems.

In high temperature application, where problems of creep, ratchetting, softening and fatigue are likely to occur, the strain prediction may form the basis for design considerations. In such a case, due to the nonlinearity of the constitutive relations, the range of applicability of small strain theory can no long be predicted. Therefore, one must resort to the use of finite strain theories. Moreover, due to the nature of the loads, high strain rates are likely to occur. This also necessitates the use of finite strain theories.

It is well established that finite deformations of a solid body particularly at elevated temperature and/or high strain rates represent coupled thermomechanical processes, which require the simultaneous solution of the coupled balance of momentum and energy equations. A proper development and solution of such thermomechanical problems requires: (1) adoption of the rational theory of thermodynamics, (2) reliable unconstrained strain and deformation kinematics, (3) a set of comprehensive elastovisco-plastic constitutive equations which accounts for the strain rate, temperature and other effects, and (4) compatibility with the available numerical tools, particularly the finite element method. These requirements taken together have not been used extensively by researches in dealing with the coupled thermomechanical problems. However, because of the need for stringent accuracy when solving practical thermomechanical problems, the importance of these requirements is being recognized.

2.4 Computational Methods

On the computational side there is a strong need to develop an algorithmic solution strategy which will enable handling the positive/indefinite stiffness characteristics associated with the passing from the pre- to the post-buckling response of structures, subjected to complex thermomechanical loads. This algorithm must be able to handle both kinematic and thermal/mechanical type material nonlinearities including inelastic effects. Also it must handle the possibility of general thermomechanical boundary conditions.

A multitude of studies have been reported on the isothermal simulation of problems where the kinematic and/or material nonlinearity is excited. In contrast, much less work is available for nonisothermal versions of such problems. This is an outgrowth of several main factors, namely:

(a) Unlike mechanical type loads which are generally applied at specific points around a given structure, transient thermally induced loads occur at every body point causing complex distributed loading and unloading fields which typically induce difficulties in simulating proper inelastic type behavior;

(b) Since thermal loads are internally induced, for nonlinear situations, it is typically quite difficult to adequately forecast the level of incrementation necessary for nonlinear equation solvers to yield converged solutions without involving an expensive time consuming trial and error procedure.

(c) For problems with highly nonlinear kinematic behavior, little is understood of the process of thermomechanical interaction; and lastly,

(d) Thermomechanically induced pre- and post-buckling behavior exhibits indefinite stiffness characteristics; such behavior precludes the use of the classical form of the incremental Newton-Raphson Scheme, which is restricted to problems with a given definiteness.

3. STATE-OF-THE-ART

3.1 - Aerothermoelasticity

The analysis of hot, fluid- induced deformation is accomplished today in a step by step approach. First, the heating rates and the aerodynamic loads are predicted by computational fluid dynamics. The governing equations are highly nonlinear and coupled. Finite difference methods dominate the scene primarily due to their simplicity, and consequently one has to pay a price for this simplicity. Extremely dense grids are required near boundaries and in regions of high velocity gradients. Varying requirements of discretization in the flow field, boundary layer, etc. can often lead to stability and accuracy problems. Finite element methods have also been applied (see for review [5-7]), but it seems that they also suffer from the same restrictions.

Then, the structural temperatures are determined using a network- type thermal analysis or via a finite element thermal analysis. Here the recent emphasis is on more integrated methods for thermal-structural analyses [8]. The "integrated" approach is based on using the same geometric model with a common nodal discretization for both analyses, although the thermal and structural models can employ different elements to suit their different requirements. Finally, structural deformations and stresses are computed by a finite element method using temperature as input data.

Two new approaches for improving the "integrated" finite element thermal- structural analysis have been suggested in [9]. The first approach is based on applying the hierarchical concept of finite element approximation to both the thermal and structural analyses. In a hierarchical approach, the accuracy of the finite element approximation is improved for the same mesh by increasing the order of interpolating functions and introducing additional unknowns via nodeless variables. The hierarchical approach to integrated thermal-structural analysis uses a common discretization for the thermal and structural analyses and seeks improvements in the accuracy of the thermal and structural analyses by independently refining the solutions using hierarchical interpolation functions for successive analyses. A key step in coupling the analyses is to use the converged temperature distribution to compute the finite element equivalent thermal forces. The second approach, called a nodeless parameter approach, uses a common discretization for both analyses and uses hierarchical interpolation functions to converge the thermal solution. The structural analysis is based on using new temperature-dependent displacement interpolation functions that have element temperatures as parameters.

The hierarchical approach offers the greatest potential for developing a general integrated thermal-structural analysis method. Maximum flexibility is permitted for independently improving the finite element approximation for each analysis, while maintaining a common discretization, and the analyses can be consistently coupled through the equivalent thermal forces. Additional study is needed to: (1) gain experience with higher order interpolation functions, (2) develop error estimation techniques to quantify convergence, and (3) study computer implementation techniques. The nodeless parameter approach offers the advantage of improving the accuracy of the structural analysis without adding unknowns. The approach, however, needs additional development before it can be implemented for general elements.

The importance of a true coupling of flow, thermal and structural analyses has been recognized only very recently by NASA [1]. Aerothermal loads on spherical dome protuberances have been studied both computationally [10] and experimentally [11]. The computations and experiments show that heating rates are augmented on windward surfaces, and that the increase in heating rates depends on the protuberance height as compared to the boundary layer thickness. The computational and experimental determinations of the augmented heating rates are based on assumed surface configuration and neglect flow-structural deformation interactions.

A simplified finite element approach for coupling flow thermal and structural analyses of aerodynamically heated panels was suggested in [12]. A boundary element method for this kind of problems is yet another suggestion [13].

3.2 Constitutive Relations

Due to the stringent design requirements for aerospace or nuclear structural components, considerable research interests have been generated on the development of constitutive models for representing the inelastic behavior of metals at elevated temperatures. In particular, a class of unified theories (or viscoplastic constitutive models) have been proposed [14-32] to simulate material responses such as cyclic plasticity, rate sensitivity, creep deformations, strain hardening or softening, etc. This approach differs from the conventional creep and plasticity theories in that both the creep and plastic deformation are treated as unified time-dependent quantities. With the exception of [29], yield surfaces are not used in these approaches. The applicability of these viscoplastic constitutive theories has been investigated by several researchers. Walker[19] compared the predictive capability of several models for Hastelloy-X at 1800°F. Milly and Allen [33] provided a qualitative as well as quantitative comparison of some of the models for IN100. Another comparison is given in [34]. All these references conclude that these models generally provide adequate results for elevated isothermal conditions, they provide poor and overly-square results at low temperature, the material constants are often difficult to obtain experimentally, the resulting rate equations are "stiff" and sensitive to numerical integration, and the models do not provide any satisfactory transient temperature capability. Beek, Allen and Milly [35] have shown that all the unified viscoplastic models mentioned above can be cast in a functionally similar form (in terms of internal state variables). Another comparison was given in [36].

None of the published literature provides a thorough evaluation of current viscoplastic constitutive models with comparison to experimental response for complex input histories. Such an evaluation is difficult at present for many reasons, namely: (1) Material constants for most models are usually available only for a single material and often for a single temperature; (2) The experimental procedures given by model developers for determining material constants from experimental data are often sketchy at best; (3) Material constants for some models are often obtained by trial-and-error and are not based on experiments; and (4) There is a lack of good experimental data against which the models can be evaluated (that is, test data which are significantly different from that used to generate the material constants).

Due to the above outlined shortcomings the authors have recently proposed new constitutive relations for metals. In the frame of the phenomenological theory of non-isothermic large elastic-thermo-viscoplastic deformation "elementary processes" were treated, which may be considered as a sequence of equilibrium states. Therefore, one can correlate a description by thermodynamic state equation to the usual description of a thermo-mechanical process. This was shown in general and for a special material.

In this frame the authors have presented a complete set of constitutive relations for nonisothermal, large strain, elasto-viscoplastic behavior of metals. It was shown [37] that the metric tensor in the convected (material) coordinate system can be linearly decomposed into elastic and (visco) plastic parts. So a yield function was assumed, which is dependent on the rate of change of stress, on the metric, on the temperature and on a set of internal variables. Moreover, a hypoelastic law was chosen to describe the thermo-elastic part of the deformations.

A time and temperature dependent viscoplasticity model was formulated in this convected material system to account for finite strains and rotations. The history and temperature dependence were incorporated through the introduction of internal variables. The choice of these variables, as well as their evolution, was motivated by thermodynamic considerations.

The nonisothermal elasto-viscoplastic deformation process was described completely by "thermodynamic state" equations. Most investigators [35], [36] (in the area of viscoplasticity) employ plastic strains as state variables. The authors study [37] shows that, in general, use of plastic strains as state variables may lead to inconsistencies with regard to thermodynamic considerations. Furthermore, the approach and formulation employed in previous works leads to the conditions that all the plastic work is completely dissipated. This, however, is in contradiction with experimental evidence, from which it emerges that part of the plastic work is used for producing residual stresses in the lattice, which, when phenomenologically considered, caused hardening. Both limitations were excluded from this [37] formulation. Accuracy of the formulation was checked on a wide range of examples [38],[39]. References [37], [38] and [39] are attached herewith as Appendix A.

3.3 Formulation of The Field Equations

As mentioned before a proper development and solution of coupled thermomechanical problems requires: (1) adoption of the rational theory of thermodynamics, (2) reliable unconstrained strain and deformation kinematics, (3) a set of comprehensive elasto-viscoplastic constitutive equations, which account for the strain rate, temperature and other effects, and (4) compatibility with the available numerical tools, particularly the finite element method. These requirements taken together have not been used extensively by researchers in dealing with coupled thermomechanical problems. However, because of the need for stringent accuracy, when solving practical thermomechanical problems, the importance of these requirements is being recognized.

Inoue and Nagaki [40] and Allen [41] developed coupled thermomechanical equations with limited applications to one-dimensional problems. Ghoneim [42] presented coupled equations, without hardening effects, and applied them to a two-dimensional axisymmetric problem of compression of an end-constrained cylinder. Lehmann [43] presented a comprehensive analysis of the development of the coupled equations with application to the necking problem in a specimen subjected to the tensile test. However, for more realistic structural components and materials, formulation is needed.

A research program was initiated by the authors to investigate the behavior of structures, when acted upon by several time- and temperature-dependent external causes. The material of the structure was taken to be metallic and the external causes included thermal, mechanical and inertia loads.

The primary objective of this program was to develop a theory and solution methodology for analyzing structures with the emphasis on thermal elasto-viscoplastic phenomena.

A complete true ab-inito rate theory of kinematics and kinetics for continuum and curved thin structures, without any restriction on the magnitude of the strains or the deformation, was formulated [37]. The time dependence and large strain behavior are incorporated through the introduction of the time rates of the metric and curvature in two coordinate systems; a fixed (spatial) one and a convected (material) coordinate system. The relations between the time derivative and the covariant derivatives (gradients) have been developed for curved space and motion, so that the velocity components supply the connection between the equations of motion and the time rate of change of the metric and curvature tensors.

The obtained complete rate field equations consist of the principles of the rate of the virtual power and the rate of conservation of energy, of the constitutive relations, and of boundary and initial conditions. These formulations provide a sound basis for the development of the adopted finite element solution procedure.

One of the most challenging aspects of finite strain formulations is to locate analytical solutions with which to compare the proposed formulation. Typically, as a first problem, a large strain uniaxial test case was analyzed. The case considered examines the rate-dependent plastic response of a bar to a deformation history that includes segments of loading, unloading, and reloading, each occurring at varying strain and temperature rates. Moreover, it was shown that the proposed formulation generates no strain energy under a pure rigid body rotation. These are surely important demonstrations but they only represent a partial test because the principal stretch directions remain constant. Finally, a problem which was discussed by Nagtegaal and de Jong, and others too, as a problem which demonstrates limitations of the constitutive models in large strain formulation, is the Couette flow problem. This problem was solved as a third example. The results of these test problems [38] (see appendix A) show that:

- The formulation can accommodate very large strains and rotations.

- The formulation does not display the oscillatory behavior in the stresses of the Couette flow problem.

- The model incorporates the simplification associated with rate-insensitive elastic response without losing the ability to model rate temperature dependent yield strength and plasticity.

The problem of buckling of shallow arches under transient thermomechanical load was investigated next [39]. The developed solution scheme is capable of predicting response which includes pre-and post-buckling with thermal creep and plastic effects. The solution procedure was demonstrated through several examples which include both creep and snap-through behavior.

The need to deal with finite strain at some stage complicates considerably any thin body kinematic theory. As for small strain, there exist robust forms of the field equations that avoid ill-conditioning in all cases. When strains are large, or at least large strains must be accounted for, there are no robust forms of the field equations. The problem here is that the stress resultants are nonlinear functions of the strains. Even if these stress-strain relations could be inverted, the dual unknown approach (static-geometric pairs) that yields satisfactory results with small strains would lead to partial differential equations nonlinear in their highest derivatives.

Another important factor is that in analyses of plate- and shell-like structures, which exhibit strong gradients in stress, strain or temperature through the thickness, accuracy demands that the so-called "degenerate" 3-D element be used, as opposed to conventional plate or shell elements. The degenerate plate/shell elements currently available, however, admit aspect ratios only up to about 10, before their performance begins to degrade. As a consequence, use of finite elements is limited.

All the above mentioned facts show that a nonlinear, thermodynamically consistent theory of shells derived from 3-D continuum mechanics in a natural and comprehensive way is very much needed. In such theory all the approximations must be thrown into a postulated 2-D form of the first law of thermodynamics.

Such an effort is being initiated by the authors.The derived shell formulation, in the least restricted form, before any simplifying assumptions are imposed, has the following desirable features:

(a) The two-dimensional,impulse-integral form of the equations of motion and the Second Law of Thermodynamics (Clausius-Duhem inequality) for a shell follow naturally and <u>exactly</u> from their three-dimensional counterparts.

(b) Unique and concrete definitions of shell variables such as stress resultants and couples, rate of deformation, spin and entropy resultants can be obtained in terms of weighted integrals of the three-dimensional quantities through the thickness.

(c) There are <u>no</u> series expansions in the thickness direction.

(d) There is no need for making use of the Kirchhoff Hypotheses in the kinematics.

(e) All approximations can be postponed until the descretization process of the integral forms of the First Law of Thermodynamics.

(f) A by-product of the descent from three-dimensional theory is that the two-dimensional temperature field (that emerges) is not a through-the-thickness average, but a true point by point distribution. This is contrary to what one finds in the literature concerning thermal stresses in the shell.

3.4 Computational Methods

As mentioned in the preceding sections the thermomechanical problems involve:

(a) Large deformation kinematics including their use in pre- and post-buckling analyses;

(b) Thermoelastic-viscoplastic material behavior;

(c) Temperature dependent thermomechanical material properties;

and

(d) Time-dependent thermomechanical loads with varying combinations/interactions between the thermal and mechanical components.

In finite element analyses for this type of problems, there exist two issues of primary concern in connection with the associated geometrical and material nonlinearities: (1) convergence in solving the global (incremental) equilibrium equations, and (2) integration of the constitutive rate equations at the local material points (or element integration points). Numerically, these two issues are interrelated. On the one hand, global equilibrium cannot be achieved if the stresses calculated at local material points are grossly inaccurate. On the other hand, the constitutive relations and stresses are not representative of the material behavior if the strains computed from the nodal displacements are in error.

In view of the above discussion, one must therefore devise a combined global/ local incremental scheme for the finite element analysis. This solution strategy must bypass the before mentioned difficulties. Thermomechanically induced pre- and post-buckling behavior exhibits indefinite characteristics [44]; such behavior precludes the use of the classical form of the incremental Newton-Raphson scheme which is restricted to problems with a given definiteness [44,45]. It seems that a constrained type strategy [46-49] is best suited for the global increments for such cases. The generality of these procedures is such that both pre- and post-buckling can be handled along with arbitrary kinematic and material nonlinearity.

An appropriate integration scheme must be chosen for handling the nonlinear viscoplastic equations. Krieg [50] pointed out the existence of numerical stiff regions in viscoplastic formulation together with a discussion of potential difficulties. The stiffness of the equations originates from the nonlinear relationship and the hardening/recovery form in evolutional equations. Formal definition of the stiffness of a set of differential equations can be found in [51], where the measure of "stiffness" is given in terms of the spectra of eigenvalues obtained from the Jacobian matrix of "associated equations" system.

The accurate integration of these stiff equations can be accomplished by various means: use of small time steps, higher-order or multi-point integration schemes, subincrementation procedures [52-55], "smart" algorithms which attempt to select appropriate time steps in order to achieve the required accuracy and stability [56-57], algorithms tailored for individual constitutive theories [52,57], or combinations of these approaches. In general, the computation time required for the accurate solution of materially nonlinear problems is directly related to the numerical integration scheme used.

The selection of an appropriate time integration scheme to be used in a computer code is very important but is often based on the answers to such questions as: "What is available in the present code?", "What will work most of the time?", "What can we use that most users will understand?", "What is the cheapest and easiest to use?", and the like. The usual response given is "it depends on the problem being solved!".

3.5 Computer Codes

The finite element method has become a standard tool for numerically simulating a wide range of engineering problems using well-established technology and commercially available software. The numerical methods have been developed continuously in parallel with progress in mathematical theory and hardware technology. The solution of three-dimensional nonlinear problems still remains costly. Analyses with sufficient resolution could be prohibitively expensive without major improvements in numerical and computational methodology. A major thrust in research and development in the present decade is to refine the algorithms and approximations in order to improve the computational performance of the finite element method. A significant development in finite element code availability and performance has occurred in the last decade.

After more than a decade of development, a wide variety of these programs are currently being used in government and in industry for the practical analysis and design of structures. The number of general-purpose nonlinear finite element programs is estimated to be about fifty. In addition, several hundred special-purpose and research-oriented nonlinear finite element programs are in existence. The potential user of a nonlinear finite element program is faced with the problem of (1) getting information about (and sorting it out) existing nonlinear finite element programs; and (2) identifying the program that is best suited for his particular needs.

The analysis capabilities and user features vary considerably from one code to the other, and therefore it is often difficult to identify the proper code that meets a specific need. A number of factors which affect the selection of a code are enumerated in the succeeding paragraphs.

(1) Analysis capabilities;
These include the range of application and limitations of the code. The limitations include both those implied by the formulation aspects and numerical procedures adopted by the code, as well as those associated with the element and material library available in the code.

(2) Adequacy of user-oriented features;
For nonlinear analyses, the user's features such as automatic mesh generation, error checks, displays of original model and of various intermediate results are essential for the effective use of the analysts' time.

(3) Maintainability;
The maintenance of nonlinear analysis codes usually includes updating the computational modules, extending the capabilities of the code and improving its performance.

(4) Adequacy of user support facilities;
In addition to the printed documentation, it is desirable to have hotline consulting, user's meetings etc.

The objective of this section is to give an overview of the current capabilities of eighteen major computer programs that can be used for solution of nonlinear structural and solid mechanics problems. The capabilities of the programs surveyed are listed in tabular form. The format is similar to the one used by Noor [58] in his 1981 survey paper on computer programs for nonlinear analyses. A summary of the major features of each program and the references are given in Appendix B. It is anticipated that this format will help in the evaluation of the available codes.

The information presented herein is based on a questionnaire sent to the program developers and published works. The authors do not claim that they have working experience with all of the listed programs.

Table 1. Finite Element Systems

			ABAQUS	ADINA	ANSYS	ASAS-NL	ASKA	DIAL	JAC	LASTAN 80	MARC	MSC/NASTRAN	NEPSAP	NISA	SAMCEF	SESAM-69	STAGSC	STRAW	TEPSA	WECAN
1.	Range of Application																			
	Nonlinear Statics	Nonlinear Response	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
		Post-buckling	•	•	•	•		•	•	•	•	•	•	•		•	•			
		Nonlinear Contact			•	•	•	•	•	•	•	•	•						•	•
		Fracture Mechanics		•		•	•			•	•	•	•			•			•	•
	Nonlinear Dynamics	Nonlinear Vibrations	•	•	•					•	•	•	•	•	•	•		•		
		Transient Response	•	•	•			•		•	•	•	•	•	•	•	•	•		•
		Wave Propagation	•	•																
	Nonlinear Interaction Problems	Fluid-Structure		•	•						•	•		•				•		
		Thermal-Mechanical	•	•	•	•			•		•	•	•	•	•	•		•	•	
		Aerothermomechanical																		
		Combustion-Mechanical									•			•						
	Large Strain	Static	•	•	•		•		•	•	•		•	•			•		•	
		Dynamic	•	•	•					•	•		•	•			•	•		
2.	Formulation																			
	Fundamental Unknowns	Displacement Method	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
		Force Method													•					
		Hybrid Method	•			•					•	•	•		•					
		Mixed Method	•												•					
		Other																		
	Reference Frame	Total Lagrangian	•	•		•	•			•	•			•	•		•			•
		Updated Lagrangian	•	•	•	•			•	•	•	•	•			•			•	
		Eulerian	•					•			•									
		Rate																		
		Other																		

				ABAQUS	ADINA	ANSYS	ASAS-NL	ASKA	DIAL	JAC	LASTAN 80	MARC	MSC/NASTRAN	NEPSAP	NISA	SAMCEF	SESAM-69	STAGSC	STRAW	TEPSA	WECAN
3.	Solution Technique																				
	Nonlinear Statics	Incremental		•	•	•	•					•		•	•		•			•	
		Newton Type Methods		•	•		•	•	•		•	•	•	•		•	•	•	•		
		Constrained Methods		•								•									
		Others			•		•			•		•									•
	Nonlinear Dynamics	Modal Superposition		•	•			•	•		•	•		•	•						
		Direct Integration	Explicit	•	•	•		•	•		•	•			•	•	•	•	•		•
			Implicit	•	•			•	•		•	•	•	•	•	•		•	•		•
			Combined E/I						•							•					
		Others																			
4.	Types of Loading																				
	Concentrated Loads			•	•	•	•	•	•	•	•	•	•	•		•	•	•	•	•	•
	Line Loads			•	•	•	•	•	•	•	•	•	•	•		•	•	•	•	•	•
	Axisymmetric Loads			•	•	•	•	•	•	•	•	•	•	•		•	•		•	•	•
	Surface Loads			•	•	•	•	•	•	•	•	•	•	•		•	•	•	•	•	•
	Volume Loads			•	•	•	•	•	•	•	•	•	•	•		•	•	•		•	•
	Gravity Loads			•	•	•	•	•	•	•	•	•	•	•		•	•	•		•	•
	Initial Stress, Strain or Velocity			•	•	•	•	•	•	•	•	•	•	•		•	•	•	•		•
	Thermal Loading			•	•	•	•	•	•	•	•	•	•	•		•	•	•	•	•	•
	Centrifugal Loading					•	•			•		•	•	•		•	•			•	•
	Deformation Dependent Loads			•			•	•	•	•	•	•	•	•		•	•	•			
	Cyclic Loading			•	•			•	•	•	•	•	•	•				•		•	•
	Random Loading							•	•			•	•								
	Gyroscopic Loading											•	•			•					
	Nonproportional Loading			•	•	•	•	•	•	•	•	•	•	•		•	•	•			
	Contact Loading			•	•	•	•	•	•	•	•	•	•	•		•				•	•
	Others										•										

		ABAQUS	ADINA	ANSYS	ASAS-NL	ASKA	DIAL	JAC	LASTAN 80	MARC	MSC/NASTRAN	NEPSAP	NISA	SAMCEF	SESAM-69	STAGSC	STRAW	TEPSA	WECAN
5.	Materials Models																		
	Isotropic	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
	Anisotropic	•	•	•	•	•	•		•	•	•	•	•	•	•	•			•
	Multilayered (laminated)	•	•	•		•	•		•	•	•	•	•	•	•	•			•
	Nonhomogeneous	•	•			•	•	•	•	•	•								•
	Temperature Dependent Elastic	•	•	•	•	•	•	•	•	•	•	•	•	•			•	•	•
	Temperature Dependent Plastic	•	•	•	•	•	•	•	•	•		•	•	•			•	•	•
	Temperature Dependent Viscoplastic									•									
	Linearly Elastic	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
	Nonlinearly Elastic	•	•	•	•	•	•	•	•	•	•			•	•	•	•	•	
	Elastic-perfectly Plastic	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
	Elastic-plastic with Strain Hardening	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
	Viscoelastic	•	•	•	•					•	•	•							
	Viscoplastic (unified)									•									
	High Temperature Creep		•	•				•	•	•		•	•				•	•	•
	Others		•							•							•	•	•
6.	Support Conditions and Constraints																		
	Axisymmetric	•	•	•	•	•	•	•	•	•	•	•	•	•			•	•	•
	At Boundries	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
	At Internal Points	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•		•	•
	Prescribed Displacements	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•		•	•
	Sliding Interfaces			•	•	•	•	•		•	•	•		•		•	•		•
	Support at Contact Points	•	•	•	•	•	•	•		•	•	•	•	•				•	•
	Elastic Foundation	•	•	•	•	•				•	•	•	•	•		•			•
	Frictional Forces	•	•	•	•		•	•		•	•								•
	Multipoint Constraints	•	•	•	•	•	•			•	•	•		•	•	•		•	•
	Cyclic Symmetry		•			•					•								

Table 1 (cont'd.)

	ABAQUS	ADINA	ANSYS	ASAS-NL	ASKA	DIAL	JAC	LASTAN 80	MARC	MSC/NASTRAN	NEPSAP	NISA	SAMCEF	SESAM-69	STAGSC	STRAW	TEPSA	WECAN
7. Elements																		
3D Rod	•	•	•	•	•	•		•	•	•	•	•	•	•	•			•
3D Beam	•	•	•	•	•	•		•	•	•	•	•		•	•			•
Plane Stress	•	•	•	•	•	•		•	•	•	•	•	•	•	•	•	•	•
Plane Strain	•	•	•	•	•	•	•	•	•	•		•	•	•	•	•	•	•
Membranes in Space	•	•	•	•	•	•		•	•	•	•	•	•	•	•			•
Shear Panels	•	•	•	•	•	•			•	•	•				•			
Plates	•	•	•	•	•	•		•	•	•	•	•		•	•			•
Thin Shells	•	•	•	•	•	•		•	•	•	•	•		•	•			•
Thick Shells	•	•	•	•	•	•		•	•	•	•	•		•				•
Shells of Revolution	•	•	•		•	•			•	•	•	•	•			•		
Axisymmetric Solids	•	•	•	•	•	•	•	•	•	•	•	•	•			•	•	•
3D Solids	•	•	•	•	•	•		•	•	•	•	•	•	•				•
Discrete Stiffeners	•	•		•	•	•			•	•	•			•	•			•
Boundary Element		•																
Gap Element		•		•					•	•							•	•
Others	•		•						•	•	•						•	•
8. Other Capabilities																		
Substructuring: Repeated Use of Iden.	•	•	•	•	•	•				•	•	•	•	•	•			•
Substructuring: Mixing Linear & Nonl.	•	•	•	•	•					•			•	•	•			
Substructuring: Mixing Diff. Types		•											•		•			
Restart Capability	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Heat Transfer Capability	•	•	•	•	•				•	•		•	•	•		•	•	•
Others										•								

4. RECOMMENDATION

4.1 Aerothermoelasticity

A finite element approach for the coupling of flow thermal and structural analyses of aerodynamically heated structures is needed. The solution of the Navier-Stokes equations for high speed compressible flow and the solution for the associated thermal-structural equations by a single finite element algorithm in one integrated, vectorized program have to be constructed. The coupling of the flow, thermal, structural analyses will provide insight into some of the fundamental features of interaction of supersonic flow with the structural response of the vehicle.

4.2 Constitutive Relations

It is proposed to initiate a research effort, in order to replace the material constitution (homogeneous and isotropic-metal) by a more general one that allows the analysis of advanced composites. Moreover, there is an essential need for more complete numerical studies of this general and new formulation in order to retain and utilize its full generality and high degree of accuracy.

It is strongly suggested that there will be interaction between this research and an experimental program for obtaining basic data for constitutive laws and for verification of the theory and related solution methodology, as applied to specific structural configurations.

4.3 Formulation

The main thrust should be to develop a finite element which will be based on the general nonlinear shell formulation and nonlinear constitutive relations. This element with the accompanying computational procedures should be adjusted to a general computer code to predict the response of shell-like structures, when subjected to time-dependent thermomechanical loads with large excursions.

In order to accomplish the overall objective, as stated initially, a number of steps or tasks need be completed. These are:

Step 1: Formulation of the principle of the rate of virtual power for the shell formulation.

Step 2: Couple the general constitutive relations to shell kinematics.

Step 3: Before developing a finite element, and before numerically testing and comparing the merits of various shell approximations, some theoretical evaluation of the degree of approximation must be attempted.

Step 4: Develop a finite element for the least restrictive shell approximation. In order to accomplish this some novel ideas and procedures must be employed. These include a tensor-oriented procedure for obtaining the element, and a discretization process that involves not only the nodal degrees of freedom but also an approximation to the "shift" tensors.

Step 5: Incorporate the developed finite element and related constitutive relations into a FEM code.

Step 6: Numerically test and evaluate the developed finite element procedure.

4.4 Computational Procedures

A combined global/local incremental procedure for the solution of the geometrical and material nonlinearities must be devised. Of major importance is the maintenance of maximum algorithmic compatibility with currently available general purpose codes such as ADINA, ANSYS, MARC, NASTRAN, etc. It seems that the constrained scheme significantly enhances the numerical operating characteristics of modified incremental Newton-Raphson algorithms.

4.5 Computer Codes

It should not be expected, in complex structural analysis problems, that one can find a single computational code. The choice is problem-related and therefore commercial codes, which are written to reach as many users as possible, cannot meet highly specialized requirements. It is important, then, to well define the needed capabilities and choose a code that comes close to satisfying the needed requirements, and at the same time it possesses flexibility for making changes.

Because of the needed capability to deal with transient heat phenomena and responses, because of the expected thermo-elasto-visco-plastic material behavior, and because of the need to deal with a code that has a modular form so that future changes can easily be accommodated the MARC code seems to be the first choice. Its rich element library and the many new computational techniques further enhance its attractiveness. In spite of this apparent recommendation, there exist two more codes with attractive features that deserve considerations. These are the

ABAQUS and ADINA codes.

Regardless of the chosen code, a capability for producing modules to meet future specific requirements need to be developed. This implies that company personnel should participate in the related Rs'D activity.

5. ADDITIONAL TASKS

The purpose of this section is two-fold. First, it contains discussion on items listed on the statement of work (Appendix C) but which may not have been covered in the main body of the report; and second, it presents a discussion on needed research tasks which affect the design and manufacture of aerospace vehicles and structures.

5.1 Substructuring

The reasons for employing substructuring in structural analyses are many and the status of substructuring techniques as well as their application to analyses and synthesis procedures are summarized in [59]. The motivation for using substructuring is computational efficiency.

In linear structural analysis, substructuring can extend the size of finite element models that can be analyzed and thus achieve computational accuracy with substantially reduced computational time. Time consuming operations on large matrices are replaced with considerably faster operations on smaller matrices.

In some nonlinear analyses, by employing, an incremental procedure for the cause (step increased in the load, for instance), the response can be obtained by linear analysis for each step. In such cases, substructuring as used in linear analysis can be employed and for the same reasons.

One of the most important applications of substructing is related to the fact that in many systems, a simpler analysis model will suffice for predicting the system response over a large part of the system, while a more complex analysis model is needed over a smaller portion of the system. In these cases, substructing is a must and it definitely enhances the required computational effort. Examples of these systems include the analysis of a flawed plate to establish both crack initiation and crack propagation. In this case, over most of the plate elastic behavior (linear) may be assumed, while close to the crack plastic behavior (nonlinear) may be needed to capture the true response of the system. If a nonlinear constitute law is used for the entire region, an accurate solution can be accomplished. Computationally though, the problem is simplified through substructing to a large region which uses a simpler constitutive law and to a small region which uses the more complex constitutive law.

Similar arguments can be used for substructuring systems for which the kinematic (and material) behavior occurs in easily identified small isolated regions.

Finally, substructuring may be the only accurate analytical tool in predicting failure phenomena in laminated configurations.

When using laminates in complex structural configurations, one encounters failure mechanisms which are unique to fiber reinforced composites. Fiber failures, matrix failures, interlaminar failure and delamination are typical of the construction. Moreover, these failures, interact and are strongly affected by the local three-dimensional state of stress. Consequently, even for very thin laminates, a three-dimensional analysis is needed to predict failure, local structural behavior, and zones of influence.

Several efforts have been made, along these lines, to account for these local effects and other deficiencies through higher order or improved plate and shell (two-dimensional) theories [60,61]. The degree of success of these efforts varies, but one can come to the rapid conclusion that some of these local effects cannot possibly be explained unless one has sufficient three-dimensionality around the area of interest. Such an approach has been proposed by Soni and Pagano [62].

In order to be able to clearly predict locally important effects in laminated composites, it is necessary to use three-dimensional finite-element analysis methods. Of course, the greater the number of 3-D elements, the better the approximation. However, because of economic capacity, the number of finite elements used in a model must be restricted. One possible strategy for overcoming such a difficulty is through substructuring [59] into a local region, where three-dimensional modeling is used, and a global region (minus the local), where two-dimensional modeling is used. In this case, the two models need an appropriate interface. Such an interface can be accomplished either through the use of suitable transition elements or the use of appropriate multipoint constraints.

5.2 Structural Similitude

Aircraft, spacecraft, and space stations comprise the class of structures that require efficiency and wisdom in design, sophistication and accuracy in analysis, and numerous and careful experimental tests of components and prototypes, in order to achieve the necessary system reliability, performance and safety.

Preliminary and/or Concept Design entails the assemblage of system mission requirements, system expected performance, identification of components and their connections as well as of manufacturing and of system assembly techniques. This is accomplished through experience based on previous similar designs, and through the possible use of a model to simulate the entire system characteristics.

Detail Design is heavily dependent on information and concepts derived from the previous step. This information identifies critical design areas which need sophisticated analyses, and design and redesign procedures to achieve the expected component performance. This step may require several independent "analysis" models, which, in many instances, require component testing. Moreover, the models must be integrated and the coupling characteristics (component interactions) must be determined.

The last step in the design process, before going to production, is the verification of the design. This step necessitates the production of large components and prototypes in order to test component and system analytical predictions and verify strength and performance requirements, under the worst loading conditions that the system is expected to encounter in service. These loads, of course, regardless of their nature (static or dynamic) and of the environment, are simulated in the manufacturer's facilities. In the aircraft industry, a flight prototype is built to test predicted performance, and load levels and distributions. In the aircraft industry, in addition to full-scale tests, certification and safety necessitate large component static and dynamic testing.

Crash-worthiness aircraft testing requires full-scale tests and several drop tests of large components. The variables and uncertainties in crash behavior are so many that the information extracted from each test, although extremely valuable, is nevertheless small by comparison to the expense. Moreover, each test provides enough new and unexpected phenomena, which suggest new tests, specifically designed to explain the new observations.

Finally, full-scale and large component testing are necessary in other industries as well. Ship building, automobile and railway car construction all rely heavily on testing.

Regardless of the application, a scaled-down (by a large factor) model, which closely represents the structural behavior of the full-scale system can prove to be an extremely beneficial tool. This possible development must be based on the existence of certain structural parameter that control the behavior of a structural system when acted upon by static and/or dynamic loads. If such structural parameters exist, a scaled-down replica can be built, which will duplicate the response of the full-scale system. The two systems are then said to be structurally similar. The term, then, that best describes this similarity is "Structural Similitude".

Dimensional analysis and similarity conditions have proven to be useful tools in other fields such as fluid mechanics [63] and locomotion of animals [64]. Several books and papers have been written on the subject of dimensional analysis. A fairly complete bibliography and historical review can be found in [63]. Two or three references are cited, herein, for the sole purpose of showing that dimensional analysis is not limited to fluid mechanics. Specialized books depict the application of dimensional analysis to astrophysics [65] and to economics [66]. See also the recent book by Kline [67].

5.3 Other Tasks Affecting The Vehicle Structure

In addition, when considering vehicles such as the National Aerospace Plane (NASP) new propulsion systems are been considered. The needed new propulsion system may consist of various systems, including the ramjet, the supersonic combustion ramjet (scramjet) and the rocket. New design concepts are been developed which integrate the propulsion system into the airframe. This, of course, suggest two things. First, that the analytical tools must be capable of dealing with engine structures as well as airframe and wing structures; and second that the airframe and propulsion manufacturers need work closer than ever before, in the analysis and synthesis of the vehicle.

Moreover, especially for NASP, control surfaces pose additional structural problems because of the uncertainties associated with the aerothermomechnical loads.

Finally, new computational schemes, such as use of parallel processors will affect both the aerodynamic and structural analyses. In this case, most of the existing codes need complete overhaul and one must virtually start from scratch.

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APPENDIX A

Selected Papers

Thermodynamically Consistent Constitutive Equations for Nonisothermal Large-Strain, Elastoplastic, Creep Behavior

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The paper is concerned with the development of constitutive relations for large nonisothermal elastic-viscoplastic deformations for metals. The kinematics of elastic-plastic deformation, valid for finite strains and rotations, is presented. The resulting elastic-plastic uncoupled equations for the deformation rate combined with use of the incremental elasticity law permits a precise and purely deductive development of elastic-viscoplastic theory. It is shown that a phenomenological thermodynamic theory in which the elastic deformation and the temperature are state variables, including few internal variables, can be utilized to construct elastic-viscoplastic constitutive equations appropriate for metals. The limiting case of inviscid plasticity is examined.

Nomenclature

da	= element of area
ds	= material line element
d_{rs}	= deformation rate
$\dot{E}_{\alpha\beta}$	= strain rate
F	= yield function
f_α^A, F_α^r	= deformation gradients
g_r, G^A	= base vectors
g^{rs}, G^{AB}	= metric tensors
J	= absolute determinant of the deformation gradient
$k, \alpha_C^A, A_{CD}^{AB}$	= internal variables
$\boldsymbol{n}$	= normal to the surface
$\boldsymbol{P}$	= force
q	= specific applied heat
s	= entropy
T	= temperature
$\boldsymbol{t}$	= traction sector
t	= time
u	= specific internal energy
V	= volume
v^r	= velocity
w	= specific mechanical work
W_{rs}	= spin tensor
x^i	= inertial coordinate system
x^α	= material coordinate system
x^A	= convected coordinate system
ρ	= density
σ	= Cauchy stress tensor
τ	= Kirchhoff stress tensor
ϕ	= specific free energy
$\overset{\nabla}{\sigma}$	= Jauman stress rate
$\dot{\sigma}$	= time derivative

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Introduction

THE prediction of inelastic behavior of metallic materials at elevated temperature has increased in importance in recent years. Many important engineering applications involve the use of metals subjected to cyclic thermomechanical loads, e.g., hot section components of turbine engines, nuclear reactor components, etc. These materials exhibit substantial complexity in their thermomechanical constitution. In fact, so complex is their material response that it could be argued that without useful a priori information, experimental characterization is futile. It is, therefore, important to be able to model accurately the nonelastic behavior of metals under cyclic mechanical and thermal loading at temperature levels for which creep and recovery introduce significant response phenomena.

Under this kind of severe loading conditions, the real world of structural behavior is highly nonlinear due to the combined action of geometrical and physical nonlinearities. On one side, finite deformation (in a stressed structure) introduce nonlinear geometric effects. On the other side, physical nonlinearities arise even in small strain regimes, whereby inelastic phenomena play a particularly important role. From a theoretical standpoint nonlinear constitutive equations should be applied only in connection with nonlinear deformation measures. However, in engineering practice, the two sources of nonlinearities are separated for practical reasons, yielding at one end of the spectrum large displacement and large rotation problems and on the other end inelastic analysis in the presence of small strain.

Constitutive models for small strain in engineering literature may generally be grouped into three categories: classical plasticity, nonlinear viscoelasticity, and theories based on microstructural phenomena. Each group can be further separated into "unified" and "uncoupled" theories, where the two differ in their approach to the treatment of rate-independent and rate-dependent inelastic deformation. The uncoupled theories decompose the inelastic strain rate into a time-independent plastic strain rate and a time-dependent creep rate with independent constitutive relations describing plastic and creep behavior. Such uncoupling of the strain components provides for simpler theories to be developed, but precludes any creep/plasticity interaction. Recognizing that cyclic plasticity, creep, and recovery are not independent phenomena but rather are very interdependent, a number of "unified" models for inherently time-dependent nonelastic deformation have been developed recently.

Classical incremental plasticity theories are macrophenomenological because they base the derivation of state variables purely on experimental results without direct reference to the microstructure of the material. Most incremental plasticity theories have four major components: 1) a stress-elastic strain relation, 2) a yield function describing the onset of plastic deformation, 3) a hardening rule that prescribes the strain hardening of the material and the modification of the yield surface during plastic flow, and 4) a flow rule that defines the components of strain that is plastic or nonrecoverable. Research in this area is voluminous. For example, Zienkiewicz and Cormeau[1] developed a rate-dependent unified theory that allows for nonassociative plasticity and strain softening, but does not model the Bauschinger effect or temperature dependence. Extensions of classical plasticity to model both rate and temperature effects were presented recently by Allen and Haisler,[2] Haisler and Cronenworth,[3] and Yamada and Sakurai.[4]

In the nonlinear viscoelastic approach, the constitutive relation is expressed as a single integral or convoluted form. This type of constitutive model employs the thermodynamic laws along with physical constraints to complete the formulation. A detailed review of several existing theories is presented by Walker.[5] Walker's theory is based on a unified viscoplastic integral developed by modifying the constitutive relations for a linear three-parameter viscoelastic solid. The theory contains clearly defined material parameters, a rate-dependent equilibrium stress, and a proposed multiaxial model. An important shortcoming of Walker's theory is its failure to model transient temperature conditions. Many other nonlinear viscoelastic theories have been proposed, including those by Cernocky and Krempl,[6] Valanis,[7] and Chabache.[8]

The microphenomenological theories attempt to represent the response of polycrystalline materials in terms of various micromechanisms of deformation and failure. Various dislocation theories have been developed to predict plastic deformation in terms of dislocation interaction, slip, glide, density, etc. Most of the material models developed to date depend primarily on the number of state variables used and their growth or evolutionary laws. Many of the recent "unified" microphenomenological theories have been discussed and evaluated by Walker[9] and Chan et al.[10]

One example of a microphysically based constitutive law is an elastic-viscoplastic theory based on two internal state variables as proposed by Bodner et al.[11] These authors demonstrate the ability of the constitutive equations to represent the principal features of cyclic loading behavior, including softening upon stress reversal, cyclic hardening or softening, cyclic saturation, cyclic relaxation, and cyclic creep. One limitation of the formulation though is that the computed stress-strain curves are independent of the strain amplitude and therefore too "flat" or "square."

Miller[12] has reported research on the modeling of cyclic plasticity with "unified" constitutive equations. He also recognizes the shortcomings of many theories in predicting hysteresis loops that are oversquare in comparison to observed experimental behavior. Improvement is accomplished by making the kinematic work-hardening coefficient depend on the back stress and the sign of the nonelastic strain term. Theories that are similar in format to Miller's have been proposed by Krieg et al.[13] and Hart.[14] The models use two internal state variables to reflect current microstructure state and are based upon models for dislocation processes in pure metals. All of these constitutive theories were formulated without the use of a yield criterion. Since these models do not contain a completely elastic regime, the function that describes the inelastic strain rate should be such that the inelastic strain rate is very small for low stress levels. Theories with a yield function and a full elastic regime have been developed for the case of isotropic hardening by Robinson[15] and Lee and Zavrel[16] for both isotropic and directional hardening.

As previously noted, the quantities utilized in the small strain theory of viscoplasticity (stress, strain, stress rate, and strain rate) are defined only within the assumption of "small strains." Yet the precise definition of what constitutes "small strain" is always left unstated. Whether or not the stresses for a given case are "small" cannot be determined a priori by geometric considerations. In general, one cannot know in advance whether, for a given loading of a material, the "small-strain" assumption (always left undefined) will hold or not. The question of whether the small-strain approximations are valid is always avoided in the "small-strain" literature. Furthermore, as Hill[17] points out, the really typical plastic problems involve changes in geometry that cannot be disregarded. In many cases, for example, it is sufficient to take into account finite plastic strains and small elastic strains or vice versa. From the theoretical viewpoint, it is desirable in all cases to have a theory that intrinsically allows for both the elastic and plastic strains to be large. Such a theory, of course, must reduce to the earlier mentioned special cases, as limiting cases. Furthermore, such theories provide a check for those obtained by generalizing small-strain theories.

The mathematical theories of deformation and flow of matter deal essentially with the gross properties of a medium. Heat and mechanical work are considered as additional causes for a change of the state of the medium. The resulting phenomena in any particular material are not unrelated. Therefore, a thermodynamical treatment of the foundation of the theory of flow and deformation is appropriate and, indeed, the obvious approach. Two very different main approaches to a thermodynamic theory of a continuum can be identified. These differ from each other in the fundamental postulates upon which the theories are based. An essential controversy (a good survey of this controversy is given in Ref. 18) can be traced through the whole discussion of the thermodynamic aspects of continuum mechanics. None of these approaches is concerned with the atomic structure of the material. Therefore, they represent purely phenomenological approximations. Both theories are characterized by the same fundamental requirement that the results should be obtained without having recourse to statical or kinetic methods.

Within each of these approaches, there are two distinct methods of describing history and dissipative effects: the functional theory[19] in which all dependent variables are assumed to depend on the entire history of the independent variables and the internal variable approach[20] wherein history dependence is postulated to appear implicitly in a set of internal variables. For experimental as well as analytical reasons,[21,22] the use of internal variables in modeling inelastic solids is gaining widespread usage in current research. The main differences among the various modern theories lie in the choice of these internal variables.

Therefore, the predictive value of an elastic-viscoplastic material model for nonisothermal, large-deformation analyses depends on three basic elements: 1) the nonlinear kinematic description of the elastic-plastic deformation, 2) thermodynamic considerations, and 3) the choice of external and internal thermodynamic variables. The objective of this paper is to examine each of these elements, illustrate their interaction, and extend these considerations to model the large, nonisothermal, elastic-viscoplastic deformation behavior of metals.

Moreover, the paper deals with the phenomenological theory of elastic-viscoplastic bodies. The process inside the lattice and at the border of the crystal grains is taken as the physical background, without considering its connection to the macroscopic behavior of the material at the present.

Kinematic and Fundamental Considerations

Consider body of volume V that occupies a finite region of Euclidean space. When subjected to prescribed body forces, surface tractions, surface temperature, and surface velocities, the body undergoes motion characterized by $x^r = \chi^r(X^\alpha, t)$. The material particles of the body are identified by coordinates X^α, which are referred to as material coordinates. The relation of the material particles to the material coordinates X^α does not change in time. The places in space that the particles occupy during the motion are identified by the coordinates x^r. Functions χ^r describe the motion of the particles X^α through space. The place occupied by the body at $t = 0$ is taken as the initial configuration. In this configuration the body is assumed to be strain-free, but not necessarily stress-free.

A third coordinate system is defined by the material coordinates as they deform with the body. This system will be denoted by X^A, which are referred to as convected coordinates. The current configuration of the body with spatial coordinates x^r and convected coordinates X^A and the initial configuration of the body with material coordinates X^α will be employed in what follows. For the spatial coordinates x^r, the covariant base vectors g_r, the contravariant base vector g^r, the metric g_{rs}, and its dual g^{rs} are used. Similarly, for the convected coordinates X^A, the covariant base vectors G_A, the contravariant base vectors G^A, the metric tensor G_{AB}, and its dual G^{AB} are used. With regard to the initial configuration, the covariant base vectors $G\alpha$, the contravariant base vectors G^α, the metric tensor $G_{\alpha\beta}$, and it dual $G^{\alpha\beta}$ are used for the material coordinates X^α.

For a second-order tensor A with components A^{rs} in the spatial coordinates and components A^{AB} in the convected coordinates, the following is true:

$$A = A^{rs} g_r g_s = A^{AB} G_A G_B \tag{1}$$

The two sets of components are related to each other through

$$A^{rs} = x^r_{,A} x^s_{,B} A^{AB} \tag{2}$$

where $X^r_{,A}$ denotes the partial derivative $\partial\chi^r(X^B,t)/\partial X^A$.

For the motion, characterized by $\chi^r(X^A,t) = \chi^r(X^\alpha,t)$, we have

$$G_A = x^r_{,A} g_r \qquad G_{AB} + X^r_{,A} x^s_{,B} g_{rs} \tag{3}$$

From Eq. (3), it is seen that $\dot{G}_{AB} = 0$, where the dot denotes time material derivative. The tensor transformation equations (1) and (2) will be used extensively in what follows.

A material line element $ds = dX^\alpha G^\alpha$ in the initial configuration when subjected to motion $\chi^r(X^\alpha,t)$ is deformed into $ds = dx^r g_r$ in the current configuration. The line element dx^r is related to the line element dX^α through the deformation gradient F^r_α by $dx^r = F^r_\alpha dX^\alpha$ where

$$F^r_\alpha = \frac{\partial x^r}{\partial x^\alpha}(x^\beta, t) \tag{4}$$

The mapping defined by the deformation gradient $F = F^r_\alpha g_r G^\alpha$ allows one to shift quantities from the current configuration to equivalent, but alternate, quantities in the initial configuration. For example, the right Caushy-Green tensor $C = C_{\alpha\beta} G^\alpha G^\beta$ and the Green-St. Venant strain tensor $E = E_{\alpha\beta} G^\alpha G^\beta$, in the initial configuration are

$$\begin{aligned} ds = dS &= g_{rs} dx^r dx^s - G_{\alpha\beta} dX^\alpha dX^\beta \\ &= (g_{rs} F^r_\alpha F^s_\beta - G_{\alpha\beta}) dX^\alpha dX^\beta \\ &= (C_{\alpha\beta} - G_{\alpha\beta}) dX^\alpha dX^\beta = 2E_{\alpha\beta} dX^\alpha dX^\beta \end{aligned} \tag{5}$$

The components of the deformation gradient, which relate a deformed line element dX^A in the convected coordinates to the undeformed line element dX^α in the initial configuration, are given by f^A_α,

$$F^r_\alpha = x^r_{,A} f^A_\alpha \tag{6}$$

Equation (6) places in a single expression the easily confused but distinct ideas of the transformation of tensor components under a change of coordinates and a shift between the current configuration and the initial configuration as a setting for the governing equations. Truesdell and Toupin[23] and Truesdell and Noll[24] emphasizes the current configuration with the spatial coordinates and an initial configuration with material coordinates. As a result, the deformation gradient plays a prominent role in their work. Only in isolated spots do they mention convected coordinates and, then, as indirectly as possible. On the other hand, Green and Adkins[25] and Sedov[26] rely heavily on convected coordinates. Our intention here is only to tie the two together for the purpose of discussing elementary assumptions. Recently, Mendelssohn and Baruch[27] review this same point as well as additional material relevant to sound numerical formulation of finite deformation problems.

The velocity $v = v^r g_r$ of a particle X^α is defined by

$$v^r \equiv \frac{\partial x^r}{\partial t}(X^\alpha, t) \tag{7}$$

From the spatial gradient of the velocity, the deformation rate

$$d = d_{rs} g^r g^s = d_{AB} G^A G^B \tag{8}$$

is defined as

$$d_{rs} \equiv \tfrac{1}{2}(V_{r,s} + V_{s,r}) \tag{9}$$

The spin

$$W = W_{rs} g^r g^s = W^{AB} G_A G_B \tag{10}$$

is defined as

$$W_{rs} \equiv \tfrac{1}{2}(V_{r,s} - V_{s,r}) \tag{11}$$

In the initial configuration, the Green-St. Venant strain rate is the shifted deformation rate,

$$\dot{E}_{\alpha\beta} = F^r_\alpha F^s_\beta d_{rs} = f^A_\alpha f^B_\beta d_{AB} \tag{12}$$

Basic to most of the postulated models of large elastic-plastic deformation behavior is the additive decomposition of d_{rs} and $E_{\alpha\beta}$ into elastic and plastic parts,[28]

$$d_{rs} = \overset{E}{d}_{rs} + \overset{P}{d}_{rs}, \quad \dot{E}_{\alpha\beta} = \dot{\overset{E}{E}}_{\alpha\beta} + \dot{\overset{P}{E}}_{\alpha\beta} \tag{13}$$

The validity of this additive decomposition in the case of finite elastic-plastic strains has been questioned by Lee and his associates.[29-32] Lee's[29] approach is based on the total purely elastic unloading from the current state to an intermediate unstressed plasticity deformed configuration, without any reverse or other kind of plastic flow. The major point in his theory is to decouple the total elastically induced distortion and measure it from a relaxed unstressed state, which is only plastically deformed from the initial to the intermediate configuration. Accordingly, the deformation gradient F is decomposed in the form

$$F = \overset{E}{F}\overset{P}{F} \tag{14}$$

where $\overset{P}{F}$ transforms a line element from the initial configuration to the intermediate configuration and $\overset{E}{F}$ from the latter to the current configuration. The intermediate configuration is chosen in such a way so that $\overset{P}{F}$ is unaffected by the presence of rigid-body motion. The deformation rate tensors d_{sr} and d_{rs} are then defined. After some manipulations, Lee shows the following relation:

$$d_{rs} = \overset{E}{d}{}^{rs} + \overset{E}{F}_{rk}\overset{P}{d}_{kl}\overset{E}{F}{}^{-1}_{ls}\Big|_s + \overset{E}{F}_{rk}\overset{P}{W}_{kl}\overset{E}{F}{}^{-1}_{ls}\Big|_s \tag{15}$$

where the subscripts s denote the symmetric parts. Generalization of Lee's theory for anisotropic elasticity was given by Mandel.[13]

Lee's theory is based on the assumption that the elastic law does not change with the history of deformation and, hence, a total elastic unloading can take place. However, it has been shown[34] that after a fair amount of plastic flow has taken place, reverse plastic deformation will result soon upon unloading, even for small strains. Therefore, a total elastic unloading cannot have any physical significance. In view of this, the theory of Lee appears as a special case of the theory of Green and Naghdi.[35] Although not as general as the theory of Green and Naghdi, Lee's theory has the advantage of being more easily fitted with the physical property of invariance of elasticity with respect to plastic deformation. In particular, Mandel[33] has pointed out that the Green-Naghdi theory is not convenient if one wants to include anisotropic elasticity effects. All this can be avoided by the use of the convected coordinates, as proposed by Sedov[26] and Lehmann.[36] The formulation presented herein will follow the work of Lehmann.

All quantities from here on will be related to the metric of the coordinate system X^A in the deformed state. Hence,

$$
\begin{aligned}
f^{\alpha}_{C} &= G^{\alpha\beta}G_{BC}\\
(f^{-1})^{A}_{\gamma} &= G^{AB}G_{\beta\gamma}
\end{aligned} \tag{16}
$$

and the deformation rate is

$$
\begin{aligned}
d^{A}_{r} &= \tfrac{1}{2}G^{AB}\dot{G}_{\beta\gamma} = -\tfrac{1}{2}G_{CB}\dot{G}^{\beta\alpha}\\
&= \tfrac{1}{2}(f^{-1})^{A}_{\beta}(\dot{f})^{\beta}_{\cdot\gamma} = -\tfrac{1}{2}(\dot{f}^{-1})^{A}_{\cdot B}f^{B}_{\gamma}
\end{aligned} \tag{17}
$$

The deformation gradient may be split into its elastic and its plastic components in the following manner:

$$
\begin{aligned}
f^{\alpha}_{C} &= \underbrace{G^{\alpha\beta}\,G_{\beta\Gamma}}_{\overset{P}{F}{}^{\alpha}_{\cdot\Gamma}}\,\underbrace{G^{\Gamma D}G_{DC}}_{\overset{E}{f}{}^{\Gamma}_{C}}\\
(f^{-1})^{A}_{\gamma} &= \underbrace{G^{AB}\,G_{B\Gamma}}_{\overset{E}{(f^{-1})}{}^{A}_{\Gamma}}\,\underbrace{G^{\Gamma\delta}\,G_{\delta\gamma}}_{\overset{P}{(f^{-1})}{}^{\Gamma}_{\gamma}}
\end{aligned} \tag{18}
$$

The use of capital greek subscripts and superscripts ($G_{\beta\Gamma}$) denotes parameters belonging to a fictitious intermediate state, which is in general incompatible. The circumstance of the noncontinuous configuration in the unstressed state has been observed by Sedov,[26] who points out that convected coordinates, as used herein, become non-Euclidean in this configuration.

This multiplicative splitting of the metric change in the convected coordinates leads to an additive splitting of the deformation rate according to

$$
\begin{aligned}
d^{A}_{C} &= \mathrm{sym}\tfrac{1}{2}\{\overset{E}{(f^{-1})}{}^{A}_{\Gamma}(\dot{\overset{E}{f}})^{\Gamma}_{\cdot C}\} + \mathrm{sym}\tfrac{1}{2}\{(f^{-1})^{A}_{\delta}(\dot{\overset{P}{f}})^{\delta}{}_{\Gamma}\overset{E}{f}{}^{\Gamma}_{C}\}\\
&= \mathrm{sym}\tfrac{1}{2}\{(\dot{\overset{E}{f}}{}^{-1})^{A}{}_{\Gamma}\overset{E}{f}{}^{\Gamma}_{C}\} - \mathrm{sym}\tfrac{1}{2}\{\overset{E}{(f^{-1})}{}^{A}_{\Gamma}(\dot{\overset{P}{f}}{}^{-1})^{\Gamma}{}_{\delta}f^{\delta}_{C}\}\\
&= \quad \overset{E}{d}{}^{A}_{C} \quad + \quad \overset{P}{d}{}^{A}_{C}
\end{aligned} \tag{19}
$$

In the current configuration of the body V, consider an element of area da on the surface of $\mathcal{S}$ with an outward normal $\boldsymbol{n} = \boldsymbol{n}_r g^r = n_A G^A$. If the force $\mathrm{d}\boldsymbol{P} = dP^r g_r = dP^A G_A$ is acting on this element, the fraction vector is $\boldsymbol{t} = d\boldsymbol{P}/da$. The Caushy stress,

$$\boldsymbol{\sigma} = \sigma^{rs}g_r g_s = \sigma^{AB}G_A G_B \tag{20}$$

defined, such that $\boldsymbol{t} = \boldsymbol{\sigma}\cdot\boldsymbol{n}$, which in component form (in terms of spatial coordinates) is $t^r = \sigma^{rs}n_s$. In the convected coordinates, it is $t^A = \sigma^{AB}n_B$.

It is convenient to work with the Kirchhoff stress tensor τ in the current configuration, obtained from the Cauchy stress by scaling

$$\tau^{A}_{B} = \frac{\rho_0}{\rho}\sigma^{A}_{B}J\sigma^{A}_{B} \tag{21}$$

where ρ denotes the current mass density, ρ_0 the mass density in the initial state, and J the absolute determinant of the deformation gradient at the current configuration.

The time derivative of a tensor such as stress, which is associated with the current configuration, admits infinitely many definitions, depending upon the coordinate system employed in the computation of such time derivatives. For a correct large-strain, large-rotation elastic-plastic model, the notion of "invariant stress fluxes" and "objectivity" must be introduced. A good treatment requires more space than is available here.[23] The corotational stress rate, also referred to as Jauman stress rate, will suffice for the purpose of the present discussion. Hence, in convected coordinates,

$$
\begin{aligned}
\overset{\nabla}{\sigma}{}^{A}_{B} &= \dot{\sigma}^{A}_{\cdot B} + d^{A}_{C}\sigma^{C}_{B} - d^{C}_{B}\sigma^{A}_{C}\\
\overset{\nabla}{\tau}{}^{A}_{B} &= \dot{\tau}^{A}_{B} + d^{A}_{C}\tau^{C}_{B} - d^{C}_{B}\tau^{A}_{C}
\end{aligned} \tag{22}
$$

From Eq. (21), the following relations between the various rates of Kirchoff stress and Cauchy stress are obtained:

$$
\begin{aligned}
\dot{\tau}^{A}_{B} &= \frac{\rho_0}{\rho}\dot{\sigma}^{A}_{B} + Jd^{C}_{C}\sigma^{A}_{B}\\
\overset{\nabla}{\tau}{}^{A}_{B} &= \frac{\rho_0}{\rho}\overset{\nabla}{\sigma}{}^{A}_{B} + Jd^{C}_{C}\sigma^{A}_{B}
\end{aligned} \tag{23}
$$

If a rate constitutive law is postulated between $\dot{\sigma}$ and d in finite inelasticity theories, then a potential does not exist, which is necessary in the variational or thermodynamics-based formulation of the problem. The basic difficulty lies with the d^{C}_{C} term. This is remedied by postulating a constitutive law between $\dot{\tau}$ and $\boldsymbol{d}$.

The Elastic Deformations

The present study is concerned with the structure of the constitutive relation of an elastic-viscoplastic (elastic-plastic) medium. The term elastic-viscoplastic means that the viscosity does not intervene in the elastic domain whose boundary, in particular, is well defined at every stage of the deformation. For simplicity, we assume further that the thermoelastic behavior of the body is isotropic and unaffected by inelastic deformation in the sense that the material constants characterizing the thermoelastic behavior are independent of inelastic deformation. Thus, we can obtain a unique relation between the elastic deformations represented by $\overset{E}{f}{}^A_C$, the Kirchhoff stresses τ^A_C, and the temperature T.[15,36]

$$\overset{E}{f}{}^A_C = \overset{E}{f}{}^A_C(\tau^A_C, T); \qquad \tau^A_C = \tau^A_C(\overset{E}{f}{}^B_D, T); \qquad T = T(\tau^A_C, \overset{E}{f}{}^B_D) \tag{24}$$

This function may be transformed into an incremental relation by differentiation with respect to time. This leads to a general expression of the form

$$\overset{E}{d}{}^A_C = \overset{E}{d}{}^A_C\{\tau^A_C, \overset{\nabla}{\tau}{}^A_C, T, \dot{T}, G_{AC}, d^A_C\} \tag{25}$$

From Eq. (24) we see that the total deformation rate enters the incremental form of the thermoelastic stress-strain relations. Therefore, the thermoelastic deformation is not independent of the inelastic deformation occurring at the same time. This follows from the fact that in the integrated form of the thermoelastic stress-strain relations Eq. (24), the stresses and the strains are referred to the deformed state of the body.

In view of the present discussion and the discussion in the previous section, the hyperelastic behavior described by Eqs. (24) and (25) will be replaced by a hypoelastic law. The hypoelastic law is a path-dependent material law, since it cannot be expressed in terms of an initial and a final state; it depends on the path connecting these states. Otherwise, if we did not make such a change, it would be necessary to retain the finite deformation measure in the constitutive law. For small elastic strains, there is practically no difference between hypoelastic and hyperelastic laws, as shown, for example, by Lehmann.[36]

The above could be illustrated by the following example. From the frequently used elastic stress-strain relation,

$$\epsilon^A_C = \tfrac{1}{2}\left\{\delta^A_C - (\overset{E}{f}{}^{-1})^A_C\right\} = \frac{1}{2G}\left\{\tau^A_C - \frac{\nu}{1+\nu}\tau^B_B\delta^A_C\right\} + \alpha(T-T_0)\delta^A_C \tag{26}$$

we get

$$\overset{E}{d}{}^A_C = \frac{1}{2G}\left\{\mathrm{sym}\left[\overset{E}{f}{}^D_C(\dot{\tau})^A_{\cdot D}\right] - \frac{\nu}{1+\nu}\overset{E}{f}{}^A_C(\dot{\tau}^B_B)\right\} + \alpha\dot{T}\overset{E}{f}{}^A_C \tag{27}$$

which may be replaced by

$$\overset{E}{d}{}^A_C = \frac{1}{2G}\left\{\overset{\nabla}{\tau}{}^A_C - \frac{\nu}{1+\nu}\overset{\nabla}{\tau}{}^B_B\delta^A_C\right\} + \alpha\dot{T}\delta^A_C + \alpha\dot{T}\delta^A_C \tag{28}$$

We assume that inelastic deformation occurs if and only if

$$F(\tau^A_C, T, k, \ldots, \alpha^A_C, \ldots, A^{AB}_{CD}, \ldots) = 0 \tag{29a}$$

$$\frac{\partial F}{\partial \tau^A_C}\overset{\nabla}{\tau}{}^A_C + \frac{\partial F}{\partial T}\dot{T} > 0 \tag{29b}$$

or, for elastic-plastic material,

$$F(\tau^A_C, T, k, \ldots, \alpha^A_C, \ldots, A^{AB}_{CD}, \ldots) > 0 \tag{30}$$

for an elastic-viscoplastic material. The function F represents the yield condition that bounds the domain of pure thermoelastic behavior in the ten-dimensional space of stress and temperature. The inequality given by Eq. (29b) is the loading condition. The actual form of the yield condition for a given material is determined by a set of so-called internal parameters, which are scalars and/or tensors of even order. The current values of the internal parameters depend on the initial state of the material and the history of the thermomechanical process.

Thermodynamic Processes

In the treatment of elastic-plastic or elastic-viscoplastic deformations, we have to distinguish between the description as a thermomechanical process and the corresponding one by means of thermodynamic state equations. It is sometimes assumed that, in the case of processes which proceed through nonequilibrium states, it is fundamentally necessary to start with a description of the process.[19,24,37] Alternatively, it has been proposed that one might assume local equilibrium for the elements of a body and therefore describe the state of the elements, in general, by state equations.[38-40] The consequences of adopting these two approaches become particularly clear when considering the influence of entropy. In the description of the process, entropy is a derived quantity and in principle we can proceed without introducing it. In the description by state equations, it is, on the contrary, a necessary state value that, at least in principle, can be immediately determined. When restricting ourselves to homogeneous, quasistatical thermomechanical processes, the description by state equations can be reviewed as equivalent to that by processes.[37,41] The controversial issues will, thus, not be discussed further.

Restricting ourselves to elementary processes, we need not analyze whether the applied heat arises from heat conduction or from heat sources. For the same reason, it is not necessary, in our case, to introduce the temperature gradient in addition to the temperature or the body forces in addition to the stresses.

The first law states, under our simplifying assumptions, that the rate of the specific internal energy $\dot{u}$ is the sum of the rates of the specific mechanical work $\dot{w}$ and the specific applied heat $\dot{q}$,

$$\dot{u} = \dot{w} + \dot{q} \tag{31}$$

The rate of mechanical work is given by

$$\dot{W} = \frac{1}{\rho_0}\tau^A_C d^C_A \tag{32}$$

and may be split into an elastic and an inelastic part according to Eq. (19),

$$\dot{W} = \frac{1}{\rho_0}\tau^A_C \overset{E}{d}{}^C_A + \frac{1}{\rho_0}\tau^A_C \overset{P}{d}{}^C_A = \overset{E}{\dot{W}} + \overset{I}{\dot{W}} \tag{33}$$

The rate of inelastic work must also be split into a part $\overset{D}{\dot{W}}$, which is dissipated at once, and into another part $\overset{S}{\dot{W}}$, which represents changes in the internal state. Thus,

$$\overset{I}{\dot{W}} = \frac{1}{\rho_0}\tau^A_C \overset{P}{d}{}^C_A = \overset{S}{\dot{W}} + \overset{D}{\dot{W}} \tag{34}$$

Only $\overset{D}{\dot{W}}$ enters the entropy production

$$T\dot{S} = \dot{q} + \overset{D}{\dot{W}} \tag{35}$$

The second law of thermodynamics requires

$$\overset{D}{\dot{W}} \geq 0 \tag{36}$$

We use as thermodynamic state variables the elastic strain, represented by $\overset{E}{f}{}^A_C$, the absolute temperature T, and a number of other internal state variables $(k,\ldots,\alpha^A_C,\ldots,A^{AB}_{CD},\ldots)$ that may be scalars and tensors of even order. The choice of $\overset{E}{f}{}^A_C$ and T as state variables is based on the fact that in pure thermoelastic deformations, both quantities form a suitable set of thermodynamic state variables. The plastic strain and the total strain are unsuitable as state variables because, in general, they do not uniquely define the state of the material. A conflicting point of view has been expressed in Refs. 42-44. The remaining state variables are added for the sake of the description of the changes of the internal structure of the material.

The specific free energy (Helmholtz function) ϕ given by

$$\phi = u - Ts \tag{37}$$

must be a unique function of the thermodynamic state variables

$$\phi = \phi(\overset{e}{f}{}^A_C, T, k, \alpha^A_C, \ldots, A^{AB}_{CD}, \ldots) \tag{38}$$

Since the elastic part of the deformation, according to our assumptions, does not depend on the plastic deformation, we may divide the free energy into two different components, as

$$\phi = \overset{E}{\phi}(\overset{E}{f}{}^A_C, T) + \overset{S}{\phi}(T,\ k,\ldots,\alpha^A_C,\ldots,A^{AB}_{CD},\ldots) \tag{39}$$

where the first component $\overset{E}{\phi}$ refers to the elastic deformation and the second $\overset{S}{\phi}$ to the changes of the internal state.

From Eqs. (31), (33-35), and (37) we derive

$$\dot\phi = -s\dot T + \dot{\overset{E}{W}} + \dot{\overset{S}{W}} \tag{40}$$

Also, we obtain from Eq. (39)

$$\begin{aligned}\dot\phi = {} & \frac{\partial\overset{E}{\phi}}{\partial \overset{E}{f}{}^A_C}\overset{\nabla}{f}{}^A_C + \frac{\partial(\overset{E}{\phi}+\overset{S}{\phi})}{\partial T}\dot T \\ & + \frac{\partial\overset{S}{\phi}}{\partial k}\dot k + \ldots + \frac{\partial\overset{S}{\phi}}{\partial\alpha^A_C}\overset{\nabla}{\alpha}{}^A_C + \ldots + \frac{\partial\overset{S}{\phi}}{\partial A^{AB}_{CD}}\overset{\nabla}{A}{}^{AB}_{CD} + \ldots\end{aligned} \tag{41}$$

By comparison of Eqs. (40) and (41), we may conclude that

$$S = -\frac{\partial(\overset{E}{\phi}+\overset{S}{\phi})}{\partial T}$$

$$\dot{\overset{S}{W}} = \frac{\partial\overset{S}{\phi}}{\partial k}\dot k + \ldots + \frac{\partial\overset{S}{\phi}}{\partial\alpha^A_C}\overset{\nabla}{\alpha}{}^A_C + \ldots + \frac{\partial\overset{S}{\phi}}{\partial A^{AB}_{CD}}\overset{\nabla}{A}{}^{AB}_{CD}\ldots$$

$$\tau^A_C = \rho_0 \overset{E}{f}{}^B_C \frac{\partial\overset{E}{\phi}}{\partial\overset{E}{f}{}^A_B} \tag{42}$$

For irreversible processes, this scheme of description has to be completed by some statements about the dependence of entropy production on the thermomechanical process. Under our assumption, we need deal only with entropy production by dissipated mechanical work, in connection with inelastic deformation. Thus, we assume, in general

$$\dot{\overset{D}{W}} = C^{AB}_{CD}\tau^C_A \overset{I}{d}{}^D_B > 0 \tag{43}$$

where

$$C^{AB}_{CD} = C^{AB}_{CD}(\overset{E}{f}{}^A_C, T, k,\ldots,\alpha^A_C,\ldots,A^{AB}_{CD}) \tag{44}$$

Equations (42) and (43) are the governing equations for nonisothermal, elastic-inelastic elementary processes. The specific free energy ϕ, which determines the nondissipated work of the thermomechanical process and the quantity C^{AB}_{CD}, which governs the entropy production, must be specified according to the material behavior.

Elastic-Viscoplastic Model

Thermomechanical processes in elastic-viscoplastic bodies cannot be considered as a sequence of equilibrium states, even in the case of the elementary processes considered here. Elastic-viscoplastic deformations are associated with nonequilibrium states. One consequence of this fact is that we may get a continuation of a process without any change in the independent process variables. This occurs, for example, in the case of creep with constant stress and temperature or in the case of an adiabatic stress relaxation under constant strain. In such cases, the body moves from a nonequilibrium state to an equilibrium state.

In order to establish the constitutive relations for elastic-viscoplastic bodies, which in the limiting case becoming elastic-inviscidly plastic, we adopt the usual assumption that the stresses, which produce the inelastic deformation, may be expressed as the sum of the so-called athermal or inviscid stresses, $\bar\tau^A_C$ and the viscous overstresses $\mathring{\tau}^A_C$

$$\tau^A_C = \bar\tau^A_C = \mathring{\tau}^A_C + (\tau^A_C - \bar\tau^A_C) \tag{45}$$

This assumption by no means detracts from the "unified" concept. The rate-independent limit of viscoplastic constitutive relation was recently discussed by Travnicek and Kratochvil.[45] Hence, the total work rate can be partitioned in the following way:

$$\dot W = \underbrace{\dot{\overset{E}{W}} + \dot{\overset{P}{W}} + \dot{\overset{V}{W}}}_{\dot{\overset{I}{W}}} = \frac{1}{\rho_0}\tau^A_C\overset{E}{d}{}^C_A + \frac{1}{\rho_0}\bar\tau^A_C\overset{P}{d}{}^C_A + \frac{1}{\rho_0}\mathring{\tau}^A_C\overset{P}{d}{}^C_A \tag{46}$$

The viscous part of the work is completely dissipated. Thus, we may write

$$\dot{\overset{V}{W}} = \dot{\overset{D_v}{W}} \tag{47}$$

Regarding the plastic work, we have already stated that one part is used for changing the internal state and only the remaining part can be considered to be dissipated. Therefore, we must write

$$\dot{\overset{P}{W}} = \dot{\overset{S}{W}} + \dot{\overset{D_p}{W}} \tag{48}$$

So, we finally obtain

$$\dot W = \dot{\overset{E}{W}} + \dot{\overset{S}{W}} + \underbrace{\dot{\overset{D_p}{W}} + \dot{\overset{D_v}{W}}}_{\dot{\overset{D}{W}}} \tag{49}$$

We have assumed that the changes of the internal state of the material can be regarded as a sequence of equilibrium states. Then, the specific energy is well defined in each state of the process and we may take the usual overall statement concerning the specific free energy. In so doing, however, we must be aware of the fact that into the part $\overset{S}{\dot W}$ of the plastic work rate $\overset{P}{\dot W}$ only the athermal stress $\bar\tau^A_C$ enter, since only these stresses are involved in the plastic mechanism. For the same reason, we can introduce only the athermal stresses $\bar\tau^A_C$ into the statement concerning the dissipated plastic work $\overset{D_p}{\dot W}$. On the other hand, we have to add the dissipated viscous work $\overset{D_v}{\dot W}$ to $\overset{D_p}{\dot W}$ in order to obtain the total rate of dissipation. The different mechanisms for determining the total dissipation and their coupling have been discussed by Perzyna.[46]

We now consider an example in which the specific free energy has the following form:

$$\phi = \overset{E}{\phi}(\overset{E}{f}{}^A_C,\ T) + \overset{S}{\phi}(T,\ k,\ \alpha^A_C) = \overset{E}{\phi}(\overset{E}{f}{}^A_C,\ T) + k + f(T) + k\alpha^A_C\alpha^C_A \tag{50}$$

In this equation, h denotes a constant with the dimension of a specific energy like the variable k and the function $f(T)$.

Furthermore, we assume that the dissipation is given by

$$\overset{D_p}{\dot W} = \frac{1}{\rho_0}\xi(\bar\tau^A_C - c\rho_0 h\alpha^A_C)\overset{P}{d}{}^C_A$$

$$\overset{D_v}{\dot W} = \frac{1}{\rho_0}(\tau^A_C - \bar\tau^A_C)\overset{P}{d}{}^C_A \tag{51}$$

where $\xi < 1$ and c denotes constant numbers. This leads to

$$\overset{D}{\dot W} = \overset{D_p}{\dot W} + \overset{D_v}{\dot W} = (\xi - 1)\overset{P}{\dot W} - \xi c h\alpha^A_C \overset{P}{d}{}^C_A + \overset{I}{\dot W} \tag{52}$$

Hence, we obtain

$$\overset{S}{\dot W} = \overset{I}{\dot W} - \overset{D}{\dot W} = (1 - \xi)\overset{P}{\dot W} + \xi c h\alpha^A_C d^C_A \tag{53}$$

On the other hand, from Eqs. (42) and (50) we have

$$\overset{S}{\dot W} = \dot k + 2h\alpha^A_C \overset{\nabla}{\alpha}{}^C_A \tag{54}$$

Equations (53) and (54) are compatible, for instance, if we put

$$\dot k = (1 - \xi)\overset{P}{\dot W} \tag{55}$$

and

$$\overset{\nabla}{\alpha}{}^C_A = \tfrac{1}{2}c\xi \overset{P}{d}{}^C_A \tag{56}$$

From Eq. (55) it follows that, in our case, the plastic work $\overset{P}{W}$ is equivalent to the thermodynamic state variable k. This is still true if we take ξ as a function of k. But it does not hold in the general case when ξ also depends on the other state variables T and α^A_C. Equation (56) shows that only in a very special case, a very unrealistic one, the state variable α^A_C is equivalent to the plastic deformation.

From the thermodynamical considerations, it follows then that we may introduce the quantities k and α^A_C, defined by Eqs. (55) and (56) or any other equivalent set $(\overset{P}{W}, c\rho_0 h\alpha^A_C)$, as internal variables into the corresponding constitutive equations of the process description.

The constitutive equations themselves are not yet determined completely by Eqs. (50), (51), (55), and (56). These given only the restrictive frame for the formulation of these equations. We may derive a complete set of constitutive equations, which is compatible with this frame, by the further assumptions:

1) The introduction of a yield condition of the form,

$$F = (\bar t^A_C - c\rho_0 h\alpha^A_C)(\bar t^C_A - c\rho_0 h\alpha^C_A) - g^2(\overset{P}{W}, T) = 0 \tag{57}$$

where $\bar t^A_C$ denotes the deviator of the Kirchhoff stresses $\bar\tau^A_C$.

2) The plastic deformation obeys the so-called normality rule,

$$\overset{P}{d}{}^A_C = \dot\lambda\frac{dF}{d\bar\tau^A_C} \tag{58}$$

3) The relations between the viscous stresses and the inelastic deformation rate are of the form,

$$\overset{P}{d}{}^A_C = \frac{1}{2\eta}\overset{*}{t}{}^A_C = \frac{1}{2\eta}(t^A_C - \bar t^A_C) \tag{59}$$

4) The quantities ξ and c are constant.

We can eliminate the athermal stresses $\bar\tau^A_C$ (which are not state variables) from the equations of evolution by considering that the inelastic deformation can be expressed in two different ways. In one, the plastic mechanism is considered and the viscous mechanism in the second. From Eq. (57), we then obtain

$$\overset{P}{d}{}^A_C = 2\dot\lambda(\bar t^A_C - c\rho_0 h\alpha^A_C) \tag{60}$$

while from Eq. (59), we have

$$\overset{P}{d}{}^A_C = \frac{1}{2\eta}(t^A_C - \bar t^A_C)$$

$$= \frac{1}{2\eta}\{t^A_C - c\rho_0 h\alpha^A_C - (\bar t^A_C - c\rho_0 h\alpha^A_C)\} \tag{61}$$

By comparing these equations for $\overset{P}{d}{}^A_C$, we get

$$\dot\lambda = \frac{1}{4\eta}\left\{\left(\frac{(t^A_C - c\rho_0 h\alpha^A_C)(t^C_A - c\rho_0 h\alpha^C_A)}{g^2}\right)^{1/2} - 1\right\} \tag{62}$$

Following the course of the process in each state, the internal parameters $\overset{P}{\dot W}$ and α^A_C and, therefore, also $k^2 = k^2(\overset{P}{W}, \alpha^A_C)$ are known. Thus, we may calculate $\dot\lambda$ from Eq. (62) and then all the other needed quantities such as $\bar t^A_C$ and $\overset{P}{d}{}^A_C$.

Discussion

Many thermodynamic considerations of nonisothermal, elastic-viscoplastic deformations refer essentially to the general fundamentals that must be observed in describing such phenomena as thermomechanical processes and then discuss what particular restrictions follow from the second law of thermodynamics. Only a few papers attempt to describe completely such processes by state equations. Most of these papers introduce plastic strains as thermodynamic state variables. But one may conclude from the consideration of the phenomena in the crystal lattice (dislocations, for example, that have completely passed through the crystal produce plastic strains but no changes of state) as well as from phenomenological observations (different states of hardening can belong to the same plastic strains) that plastic strains in general cannot be regarded as state variables. Furthermore, all these papers consider the plastic work as completely dissipated. However, this is in contradiction with experimental results, from which it emerges that one part of plastic work is used for producing states of residual stresses in the lattice, which, when phenomenologically considered, cause hardening.

The results in work presented here can be extended to more complex constitutive equations by introducing more internal parameters or state variables. We may extend our approach to more general, anisotropic hardening materials by assuming [see Eq. (50)], for example, that

$$\overset{E}{\phi} = \overset{E}{\phi}(f^A_C, T) + \overset{S}{\phi}(T, k, \alpha^A_C, A^{AB}_{CD})$$

$$= \overset{E}{\phi}(\overset{E}{f}{}^A_C, T) + k + f(T) + A^{AB}_{CD}\alpha^C_A\alpha^D_B \tag{63}$$

Also, it may be more advantageous to replace the assumption in Eq. (58) for the plastic deformation rate by

$$\overset{P}{d}{}^A_C = \dot{\lambda}\frac{\partial F}{\partial \tau^A_C} + B^{AB}_{CD}\overset{\nabla}{\tau}{}^D_B \tag{64}$$

This form of this model appears to be more suitable for representing some experimental results in which second-order effects and some deviations from the normality rule have been observed. Sometimes, the normality rule is considered as a fundamental law based on an entropy production principle. But we should keep in mind that, since not all of the plastic work is dissipated, we cannot expect the total plastic deformation rate to obey the theory of plastic potential even though the mentioned principles of entropy production are correct.

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THERMO-ELASTO-VISCOPLASTIC ANALYSIS OF PROBLEMS IN EXTENSION AND SHEAR

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Abstract

The problems of extension and shear behavior of structural elements made of carbon steel and subjected to large thermomechanical loads are investigated. The analysis is based on nonlinear geometric and constitutive relations, and is expressed in a rate form. The material constitutive equations are capable of reproducing all nonisothermal, elasto-viscoplastic characteristics. The results of the test problems show that: (i) the formulation can accommodate very large strains and rotations, (ii) the model incorporates the simplification associated with rate-insensitive elastic response without losing the ability to model a rate-temperature dependent yield strength and plasticity, and (iii) the formulation does not display oscillatory behavior in the stresses for the simple shear problem.

I. Introduction

The prediction of inelastic behavior of metallic materials at elevated temperatures has recently increased in importance. The operating conditions within the hot section of a rocket motor or a modern gas turbine engine present an extremely harsh thermo- mechanical environment. Large thermal transients are induced each time the engine is started or shut down. Additional thermal transients from an elevated ambient occur, whenever the engine power level is adjusted to meet flight requirements. The structural elements employed to construct such hot sections, as well as any engine components located therein, must be capable of withstanding such extreme conditions. Failure of components would, due to the critical nature of the hot section, lead to an immediate and catastrophic loss in power and thus cannot be tolerated. Consequently, assuring satisfactory long term performance for such components is a major concern for the designer.

Under this kind of severe loading conditions, the structural behavior is highly nonlinear due to the combined action of geometrical and physical nonlinearities. On one side, finite deformation in a stressed structure introduces nonlinear geometric effects. On the other side, physical nonlinearities arise even in small strain regimes, whereby inelastic phenomena play a particularly important role. From a theoretical standpoint, nonlinear constitutive equations should be applied only in connection with nonlinear transformation measures (implying both deformation and rotation). However, in almost all of the works in this area [1], the two identified sources of nonlinearities are always separated. The separation yields, at one end of the spectrum, problems of large response, while at the other end, problems of isothermal and nonisothermal viscous behavior in the presence of small strain.

The classical theories, in which the material response is characterized as a combination of distinct elastic, thermal, time independent inelastic (plastic) and time dependent inelastic (creep) deformation components cannot explain some phenomena, which can be observed in complex thermo–mechanical loading histories. This is particularly true when high temperature nonisothermal processes must be taken into account. There is a sizeable body of literature [1,2] on phenomenological constitutive equations for the rate– and temperature– dependent plastic deformation behavior of metallic materials. However, almost all of these new "unified" theories are based on small strain assumptions and several suffer from some thermodynamic inconsistencies.

In previous papers [3,4], the authors have presented an alternative constitutive law for elastic–thermo–viscoplastic behavior of metallic materials, in which the main features are: (a) unconstrained strain and deformation kinematics, (b) selection of reference space and configuration for the stress tensor, bearing in mind the rheologies of real materials, (c) an intrinsic relation which satisfies material objectivity, (d) thermodynamic consistency, and (e) proper choice of external and internal thermodynamic variables.

The present paper focuses on a general mathematical model and solution methodologies, to examine extension and shear behavior of structural elements made of a realistic (C 45) material and subjected to nonisothermal large elasto–plastic deformation.

II. Elasto–Thermo–Viscoplastic Constitutive Relations

In a previous works [3,4] following the ideas of Lehmann [5,6] the authors have presented a complete set of constitutive relations for nonisothermal, large strain, elasto–viscoplastic behavior of metals. It was shown [3,4] that the metric tensor in the convected (material) coordinate system can be linearly decomposed into elastic and (visco) plastic parts. So a yield function was assumed, which is dependent on the rate of change of stress, on the metric, on the temperature and on a set of internal variables. Moreover, a hypoelastic law was chosen to describe the thermo-elastic part of the deformation.

A time and temperature–dependent viscoplasticity model was formulated in this convected material system to account for finite strains and rotations. The history and temperature dependence were incorporated through the introduction of internal variables. The choice of these variables, as well as their evolution, was motivated by thermodynamic considerations.

The nonisothermal elasto-viscoplastic deformation process was described completely by "thermodynamic state" equations. Most investigators [1,2] (in the area of viscoplasticity) employ plastic strains as state variables. The author's previous study [3] shows that, in general, use of plastic strains as state variables may lead to inconsistencies with regard to thermodynamic considerations. Furthermore, the approach and formulation employed in previous works [1,2] lead to the condition that all plastic work is completely dissipated. This, however, is in contradiction with experimental evidence, from which it emerges that part of the plastic work is used for producing residual stresses in the lattice, which, when phenomenologically considered, causes hardening. Both limitations were excluded from this [3] formulation.

The constitutive relation will be rephrased here in some different form. For brevity we compile only some notations and fundamental relations which are used in the formulation of the intended constitutive law. For details, see Refs. 3 and 4.

Concerning the formulation of constitutive laws, it is advantageous to use a material (co-moving) coordinate system. The transformation (f^i_k) from the undeformed state (metric $\overset{\circ}{g}{}^{ir}$) to the deformed state (metric g_{rk}) can be represented by the tensor:

$$f^i_k = \overset{\circ}{g}{}^{ir} g_{rk} \quad or \quad (f^{-1})^i_k = g^{ir} \overset{\circ}{g}_{rk} \tag{1}$$

The total deformation rate is defined by

$$d^i_k = \frac{1}{2} g^{ir} \dot{g}_{rk} = -\frac{1}{2} g_{ir} \dot{g}^{rk} = \frac{1}{2} (f^{-1})^i_r (\dot{f})^r_{.k} = -\frac{1}{2} (\dot{f}^{-1})^i_{.r} f^r_k \tag{2}$$

here ($\dot{}$) denotes time material derivative. The expression

$$\overset{\nabla}{f^i_k} = (\dot{f})^i_{.k} + d^i_r f^r_k - d^r_k f^i_r = sym\{(\dot{f})^i_{.k}\} \tag{3}$$

represents the symmetric part of $(\dot{f})^i_{.k}$ or the covariant derivative with respect to time, also called the convected derivation, which is due to Zaremba and Jaumann. [5]

The total deformation can be decomposed according to

$$f^i_k = \overset{\circ}{g}{}^{im} \overset{*}{g}_{mr} \overset{*}{g}{}^{rs} g_{sk} = \overset{(i)}{f^i_r} \overset{(r)}{f^r_k} \tag{4}$$

where the superscript $(\overset{*}{\ })$ relates to a fictitious configuration defined by a fictitious reversible process with frozen internal variables. The decomposition of Eq. (4) leads to an additive decomposition of the deformation rate

$$d^i_k = \overset{(r)}{d^i_k} + \overset{(i)}{d^i_k} \tag{5}$$

where $\overset{(r)}{d^i_k}$ is related to the reversible deformations, and $\overset{(i)}{d^i_k}$ denotes the remaining part of the deformation rate.

For the description of the stress state, we introduce the Kirchoff stress tensor s^i_k, which is connected with the real Cauchy stress tensor σ^i_k, by the relation:

$$s^i_k = \frac{\overset{\circ}{\rho}}{\rho} \sigma^i_k \tag{6}$$

where ρ is the mass density.

Assuming that the elastic behavior is not affected essentially by inelastic deformations, the following hypoelastic incremental law was chosen [3]

$$\overset{(r)}{d^i_k} = \frac{1}{2G} \overset{\nabla}{t^i_k} + \{\frac{1}{9K} \dot{s}^r_r + \alpha \dot{T}\} \delta^i_k \tag{7}$$

with

- t^i_k : weighted stress deviator
- G : shear modulus
- K : bulk modulus
- α : coefficient of thermal expansion

The following constitutive relations were established [3] for the inelastic behavior.
yield condition:

$$F = (t^i_k - c\,\overset{\circ}{\rho}\,g\beta^i_k)(t^k_i - c\,\overset{\circ}{\rho}\,g\beta^k_i) - k^2(A,T) = f^2 - k^2 > 0 \tag{8}$$

accompanying equilibrium state:

$$\overline{F} = (\bar{t}^i_k - c\,\overset{\circ}{\rho}\,g\beta^i_k)(\bar{t}^k_i - c\,\overset{\circ}{\rho}\,g\beta^k_i) - k^2(A,T) = \overline{f}^2 - k^2 = 0 \tag{9}$$

evolution law for inelastic deformations:

$$\overset{(i)}{d^i_k} = 2\dot{\lambda}(\bar{t}^i_k - c\,\overset{\circ}{\rho}\,g\beta^i_k) \tag{10}$$

with

$$\dot{\lambda} = \frac{1}{4\eta}\left(\sqrt{\frac{(t^i_k - c\,\overset{\circ}{\rho}\,g\beta^i_k)(t^k_i - c\,\overset{\circ}{\rho}\,g\beta^k_i)}{k^2}} - 1\right) \tag{11}$$

and

$$\bar{t}^i_k = \frac{1}{1+4\eta\dot{\lambda}}(t^i_k - c\,\overset{\circ}{\rho}\,g\beta^i_k) + c\,\overset{\circ}{\rho}\,g\beta^i_k \tag{12}$$

evolution laws for the internal variables:

$$\dot{A} = \frac{1}{\overset{\circ}{\rho}}\bar{t}^i_k\,\overset{(i)}{d^k_i} \tag{13}$$

$$\overset{\nabla}{\beta^i_k} = \xi\,\overset{(i)}{d^i_k} \tag{14}$$

if

$$F = 0 \quad and \quad \frac{\partial F}{\partial s^i_k}\,\overset{\nabla}{s^i_k} + \frac{\partial F}{\partial T}\dot{T} > 0 \tag{15}$$

then

$$d^i_k = \overset{(r)}{d^i_k} \tag{16}$$

$$\overset{(i)}{d^i_k} = 0 \quad and \quad \overset{\nabla}{d^i_k} = 2\ddot{\lambda}(t^i_k - c\,\overset{\circ}{\rho}\,g\beta^i_k) \tag{17}$$

with

$$\ddot{\lambda} = \frac{1}{8\eta k^2}\{2(t^i_k - c\,\overset{\circ}{\rho}\,g\beta^i_k)\,\overset{\nabla}{t^k_i} - \frac{\partial k^2}{\partial T}\dot{T}\} \tag{18}$$

if

$$F = 0 \quad and \quad \frac{\partial F}{\partial s^i_k}\,\overset{\nabla}{s^i_k} + \frac{\partial F}{\partial T}\dot{T} \leq 0 \tag{19}$$

or ,if

$$F < 0 \tag{20}$$

then

$$\begin{aligned} d^i_k &= \overset{(r)}{d^i_k} \\ \dot{A} &= 0 \\ \overset{\nabla}{\beta^i_k} &= 0 \end{aligned} \tag{21}$$

Within the developed frame, the elasto–viscoplastic behavior is governed by the scalar material functions $c(s^i_k, T, A, \beta^i_k)$, $k^2(A,T)$, $g(s^i_k, A, T, \beta^i_k)$, $\xi(A, T, \beta^i_k)$, and $\eta(A, T, \beta^i_k)$. These material functions can be determined from a series of monotonic and cyclic processes with proportional and nonproportional paths at different temperature levels [4].

III. General Formulation

The rate form of the constitutive equations suggests [7,8] that a rate approach be taken toward the entire problem so that flow is viewed as a history dependent process rather than an event. For this purpose, a complete true ab-initio rate theory of kinematics and kinetics for the continuum and curved thin structures, without any restriction on the magnitude of the transformation was presented in Ref. 4. It is implemented with respect to a body– fixed system of convected coordinates, and it is valid for finite strains and finite rotations. The time dependence and large strain behavior are incorporated through the introduction of the time rates of change of the metric (d_{ik}) and of the spin (ω_{ik}). The constitutive law may be applied to the conservation of momentum via an appropriate variational principle. The principle of virtual power (or of virtual velocities) reads

$$\int_V \sigma^{ij}\delta v_{j,i} dV - \int_V \rho f^j \delta v_j dV - \int_A \nu T^j \delta v_j dA = 0 \tag{22}$$

where δv_j are the virtual velocities, f^j the body forces per unit mass and νT^j the surface tractions. Total differentiation of Eq. (22) yields,

$$\begin{aligned}\int_V \left(\frac{d\sigma^{ij}}{dt} + \sigma^{ij} d^k_{.k} - v^i_{,k}\sigma^{kj}\right)\delta v_{j,i} dV - \int_V \rho \frac{df^j}{dt}\delta v_j dV \\ - \int_A \nu \frac{dT^j}{dt}\delta v_j dA + \int_V \sigma^{ij}\left(\frac{d\delta v_j}{dt}\right),idV - \int_V \rho f^j \frac{d\delta v_j}{dt} dV \\ - \int_A \nu T^j \frac{d\delta v_j}{dt} dA = 0\end{aligned} \tag{23}$$

At any instant, Eq. (23) must be satisfied. The virtual velocity and its time derivative are then independent. Moreover, the last three terms of Eq. (23) are equivalent to Eq. (22). Hence, the principle of the rate of virtual power may be obtained in its concise form. For further classifications, the total derivative of the stress components will be represented by the Jaumann derivative, and the following integrals are defined by

$$I_e = \int_V \overset{\nabla}{\sigma^{ij}} \; \delta v_{j,i} dV \tag{24}$$

$$I_d = \int_V (\sigma^{ij} d^k_k - \sigma^{kj} d^i_{.k})\delta v_{j,i} dV \tag{25}$$

$$I_r = \int_V \omega^j_{.k}\sigma^{kj}\delta v_{j,i} dV \tag{26}$$

Then, substitution in Eq. (23) yields the final form of the principle of the rate of virtual power,

$$I = I_e + I_d + I_r = \int_V \rho \frac{df^j}{dt}\delta v_j dV + \int_A \nu \frac{dT^j}{dt}\delta v_j dA \tag{27}$$

The quasi-linear nature of the principle of the rate of virtual power suggests the adoption of an incremental approach to numerical integration with respect to time. The availability of the field formulation provides assurance of the completeness of the incremental equations and allows the use of any convenient procedure for spatial integration over the domain V.

IV. Simplified Constitutive Relations

One of the most challenging aspects of finite strain formulations is to locate a known solution with which to compare a proposed formulation. Typically, as a first problem, a large strain uniaxial test case is analyzed. The uniaxial tensile test is a common and simple way to characterize the stress–strain relation for a given material, since the tensor components used in the constitutive relations have to be related to this uniaxial test. This example clearly demonstrates how the general constitutive relations can be applied to a particular real material. This material law is then applied to the remaining examples.

For a carbon steel C45 (DIN 1720), in a pure tension test at a moderate temperature and strain rate, the material behavior, which is shown in Fig. 1, is obtained from Ref. 9. From this we may derive the stress–stain–temperature relations for loading in pure tension in the form

$$\sigma = \sigma(\epsilon, T) \tag{28}$$

For our purpose it is more useful to write this relation in the form

$$\sigma = \sigma(A, T) = \frac{c_1(T)A}{c_2(T) + A} + \sigma_0(T) \tag{29}$$

In our special case we get

$$\begin{aligned} c_1(T) &= \quad 72.42 - 36.03 * 10^{-3}T \quad \frac{N}{mm^2} \\ c_2(T) &= \quad 7.35 - 8.04 * 10^{-3}T \quad \frac{N}{mm^2} \\ \sigma_0(T) &= \quad 47.41 - 38.9 * 10^{-3}T \quad \frac{N}{mm^2} \end{aligned} \tag{30}$$

with T in ${}^0K$.

We may consider the carbon steel approximately as an isotropic work–hardening material obeying the Von Misses–Hill yield condition. Furthermore, we assume that a constant ratio of 90% of the plastic work is dissipated. With these assumptions, we get the following desecription for the material under consideration,

independent process variables : s^i_k, T
dependent process variables : (a) A or $k^2(A,T)$
(b) $f^i_k, q, \ldots$

yield condition :

$$\begin{aligned} F(s^i_k, T, A) &= t^i_k t^k_i - k^2(A, T) = 0, \\ k^2(A, T) &= \frac{2}{3}[\sigma_0(T) + \frac{c_1(T)A}{c_2(T) + A}]^2 \end{aligned} \tag{31}$$

loading condition :

$$\frac{\partial F}{\partial s^i_k} \overset{\nabla}{s^i_k} + \frac{\partial F}{\partial T}\dot{T} = 2t^i_k \overset{\nabla}{t^k_i} - \frac{\partial k^2}{\partial T}\dot{T} > 0 \tag{32}$$

elastic strain rate :

$$\overset{(r)}{d^i_k} = \frac{1}{2G}[\overset{\nabla}{s^i_k} - \frac{\nu}{1+\nu} \overset{\nabla}{s^j_j} \delta^i_k] + \alpha\dot{T}\delta^i_k \tag{33}$$

with $\alpha = 11.9 * 10^{-6}K^{-1}$,
plastic strain rate :
when Eqs. (31) and (32) are fulfilled :

$$\overset{(i)}{d^i_k} = \dot{\lambda}\frac{\partial F}{\partial s^i_k} = \frac{2t^m_n \overset{\nabla}{t^n_m} - \frac{\partial k^2}{\partial T}\dot{T}}{k^2 \frac{\partial k^2}{\partial A}} \tag{34}$$

otherwise :

$$\overset{(i)}{d^i_k} = 0 \tag{35}$$

rate of "plastic work" :

$$\dot{A} = s^i_k \overset{(i)}{d^k_i} \tag{36}$$

rate of applied heat :

$$\dot{q} = c\dot{T} - \xi\dot{A} \tag{37}$$

with $\xi = 0.9 = const.$ and $c = 465\frac{j}{kg\,K}$ (heat capacity).

V. Uniaxial Examples

Uniaxial loading at different rates, loading–unloading– reloading, and jump tests are selected for the investigation of the response of the described model. Figure 2 presents the strain rate effect on the stress–strain curve. The results are presented for dimensionless stress σ/σ_0. It is clear that increasing the strain rate, in general, increases the yield stress and the plastic hardening.

Loading–unloading–reloading results are shown in Fig. 3. In this figure, the upper and the lower dotted curves represent the constant–rate stress–strain curves at 1.0 and 0.1 sec $^{-1}$, respectively. And the solid as well as the dashed curves represents the loading–unloading–reloading stress response. Dotted curves are plotted for the sake of comparison. Clearly, upon reloading at a different rate from that of loading–unloading, the yielding takes place at values that are influenced by the previous strain rate of loading–unloading.

The jump test results, where the strain rate is suddenly changed, are displayed in Fig. 4. As in the case of loading–unloading–reloading, the jump test results reveal the strain rate–history dependence inherited in the model. Upon decreasing the strain rate from 1.0 to 0.1 sec $^{-1}$ (solid line) the stress drops to values higher than that of the constant–rate loading at 0.1 sec $^{-1}$ (lower dotted curve). This is because the response after the jump is influenced by the previous strain rate of loading before the jump(1.0 sec $^{-1}$). Similarly, when the strain rate is increased from 0.1 to 1.0 sec $^{-1}$ (dashed line) the stress response rises to values lower than that of the constant–rate loading at 1.0 sec $^{-1}$ (upper dotted curve).

Finally, an example is considered, through which we show the response of the system corresponding to a given strain history. The history is depicted on Fig. 5(a), and all rates (positive or negative) are equal to 10 sec $^{-1}$. The response is shown in Fig. 5(b). The letters in Fig. 5(b) correspond to the letters of Fig. 5(a). The dotted lines in Fig. 5(b) were obtained under the assumption that the total plastic work was dissipated. Due to the viscid effect, the rate–dependent stress–strain curves (solid lines) have continuous slopes at the shifting points. Note that the transient hardening causes the subcycles not to be closed.

VI. Simple Shear Examples

The previous examples, although important, only represent a partial test because the principal stretch directions remain constant. The problem which was discussed by Nagtegaal and de Jong [10] and by Rolf and Bathe [11] as a problem which demonstrates limitations of the constitutive models in many finite strain formulations is the simple shear problem.

We take the same material as in the former examples, C 45, and consider a simple shear process, by effectively applying a shear strain, γ (see Fig. 6). We denote this material as material a.

The simple shear problem is defined by

$$X_1 = x_1 + KtX_2, \quad X_2 = x_2, \quad X_3 = x_3 \tag{38}$$

where the capital letters denote current position. This leads to the following nonzero components of deformation rates and spin

$$d_{12} = d_{21} = \frac{K}{2}, \quad \omega_{12} = -\omega_{21} = \frac{K}{2} \tag{39}$$

Please note that Eqs.(38) define simple shear for isothermal material behavior (process). This case was treated in Refs 10 and 11 and herein also. In adiabatic material behavior (process), X_3 must contain a temperature effect, which in our formulation appears through the coupling of (thermal) deformation and material behavior [constitutive law, see Eq.(33)]. The angular velocity of a line of material points depends only on its current orientation, angle γ, and is given by

$$\dot{\gamma} = -K cos^2 \gamma \tag{40}$$

Note that $K/2$ is the magnitude of the angular velocity of the material lines, when $\gamma = 0$ and $\gamma = \pi$, which instantaneously coincide with the principal directions of the rate of deformation tensor, d_{ij}. This is also the average of the angular velocities over all directions in the current configuration.

The process will be carried out, on the one hand, isothermally and, on the other hand, adiabatically. We find the solution of the problem by numerically integrating a system of first–order differential equations, originating from Eqs. (31) – (37). In the first case (isothermal process), the total deformation rate d^i_k and the temperature T_0 are given, and in the second case (adiabatic process) the total deformation rate is prescribed, and there is no heat transfer.

For comparison, we introduce, furthermore, a theoretical material whose yield condition is unaffected by temperature. This means, for this material, the hardening parameter k^2 is

$$k^2 = k^2(A, T_0)$$

In isothermal processes this material (denoted as material b) shows the same behavior as material a. But in adiabatic processes we have differences. For material a, the temperature influences the yield condition as well as the elastic part of deformation. For material b, only the elastic components of the deformation are affected by temperature changes.

Regarding this we must distinguish three cases:

(I) : isothermal processes with material a or b,

(IIA) : adiabatic processes with material a (hardening rule depending on temperature),

(IIB) : adiabatic processes with material b (hardening rule independent of temperature).

The results for the shear stresses and the temperature are shown in Fig. 7. We see that the differences between the shear stresses in the isothermal and in the adiabatic processes are mainly influenced by the dependence of the yield condition upon temperature. The differences between cases (I) and (IIB) are negligible, but not the differences between (IIA) and (IIB). It should be remarked that in case (IIA), we get a maximum shear stress for $\vartheta = 0.87$. So for larger deformation we find in this case a softening effect due to the increasing temperature. With respect to the temperature the differences between the adiabatic cases, (IIA) and (IIB) are rather small, since the differences in the plastic work, in both these cases, are not so important.

The second-order effects are more influenced by the temperature than the first-order effects. This can be seen from Fig. 8. The effects are partially changed in the opposite direction (see stress σ_{11}). This is due to the strong influence of the temperature on the elastic deformation. We may conclude this from the fact, that the differences between cases (IIA) and (IIB) are less than the differences between cases (I) and (IIA) or (IIB).

It is clear that the importance of the issue discussed here is not limited to the simple shear example, but would be of consequence in problems whenever the local shearing strain at a point or points in a structure becomes large during deformation of the structure.

VII. Conclusion

A true ab-initio rate theory has been developed by the present authors [3,4] to deal and predict the behavior of structural components made of metallic materials, when acted upon by large time-dependent thermo-mechanical loads. This rate theory is based on coupled material and deformation behavior in a highly nonlinear form, which also includes time and temperature effects.

The main thrust of this paper is to demonstrate the capabilities of this formulation, through problems of extension and shear. Among the most important findings of the test problems one may list the following: (i) the formulation can accommodate very large strains and rotations, (ii) the model incorporates the simplification associated with rate-insensitive elastic response without losing the ability to model a rate-temperature dependent yield strength and plasticity, (iii) the general constitutive relations can be applied to a particular real material, (iv) the constitutive relations are sensitive to strain rate and temperature history, (v) the formulation does not display oscillatory behavior in the stresses for the simple shear problem, and (vi) the influence of the thermo-mechanical coupling on the processes can become very large and must be considered.

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Figure Captions

Fig. 1. Carbon Steel C 45 in Tension

Fig. 2. Effect of Strain Rate on the Stress–Strain Curve

Fig. 3. Loading–Unloading–Reloading at Different Strain Rates

Fig. 4. Jump Test

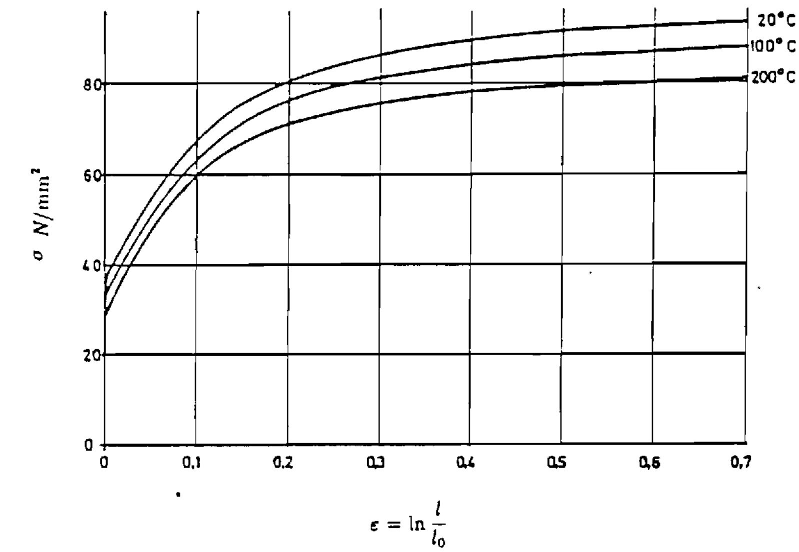
Fig. 5(a). Loading Program of Variable Strain Amplitude ($d = 10sec^{-1}$)

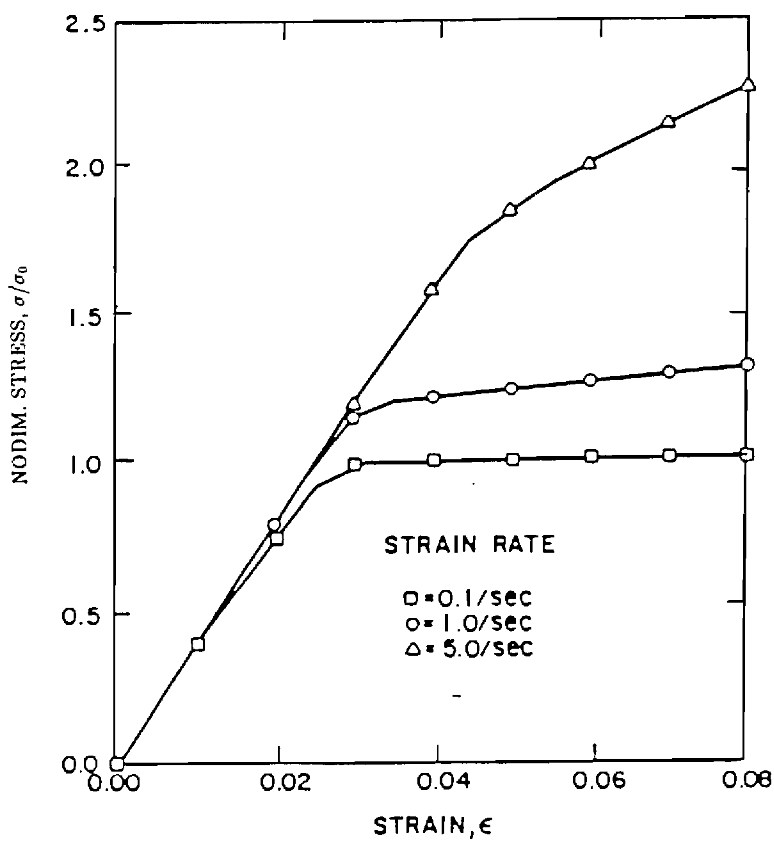
Fig. 5(b). Stress–Strain Response

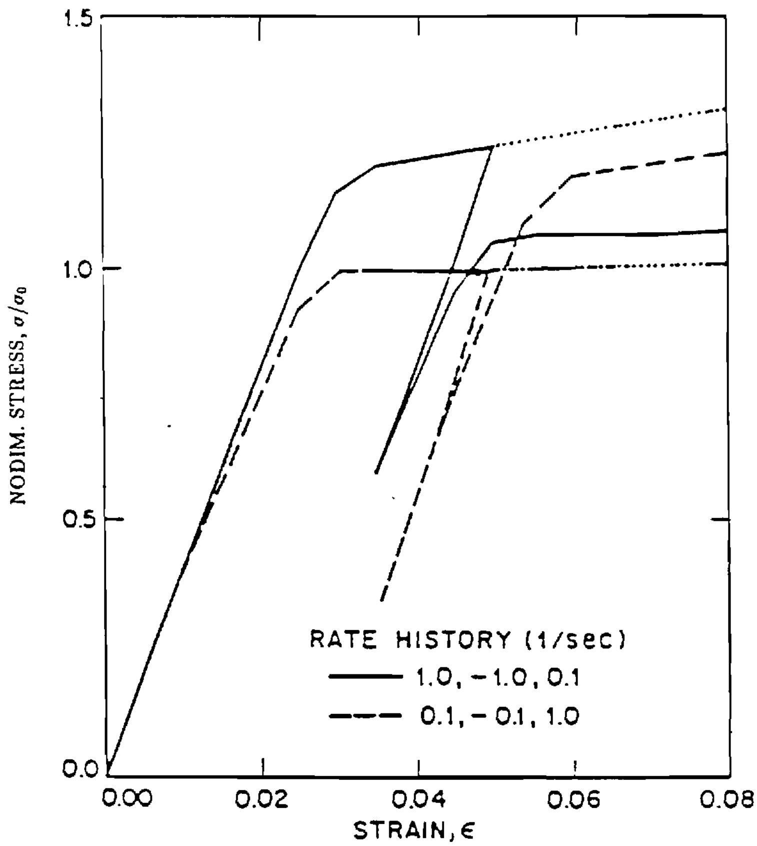
Fig. 6. Simple Shear

Fig. 7. Response to Simple Shear Test

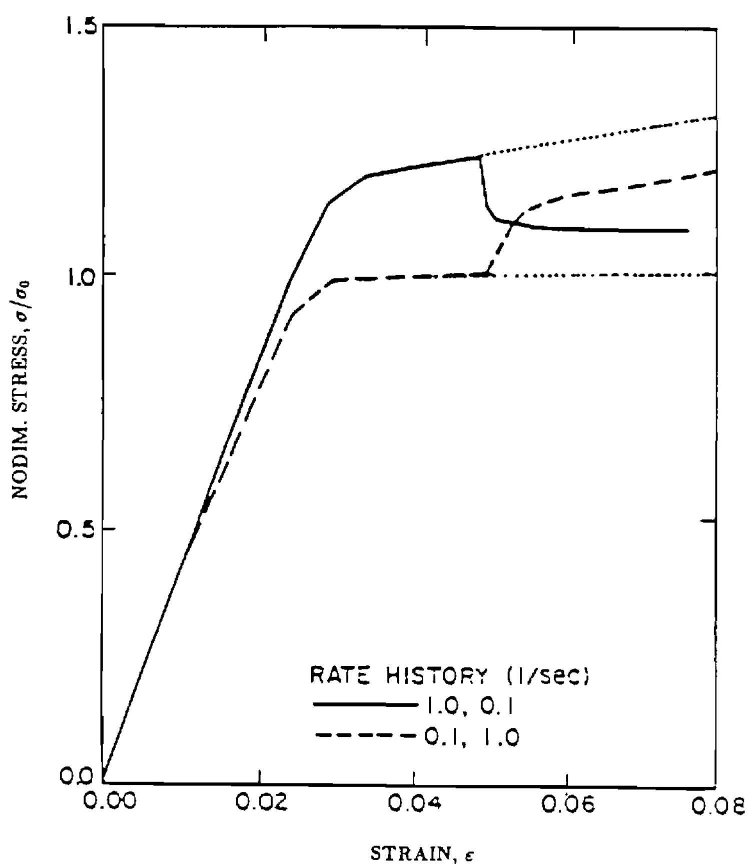
Fig. 8. Response to Simple Shear Test


20°C
100°C
200°C
σ N/mm²
0
20
40
60
80
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
$\epsilon = \ln \frac{l}{l_0}$

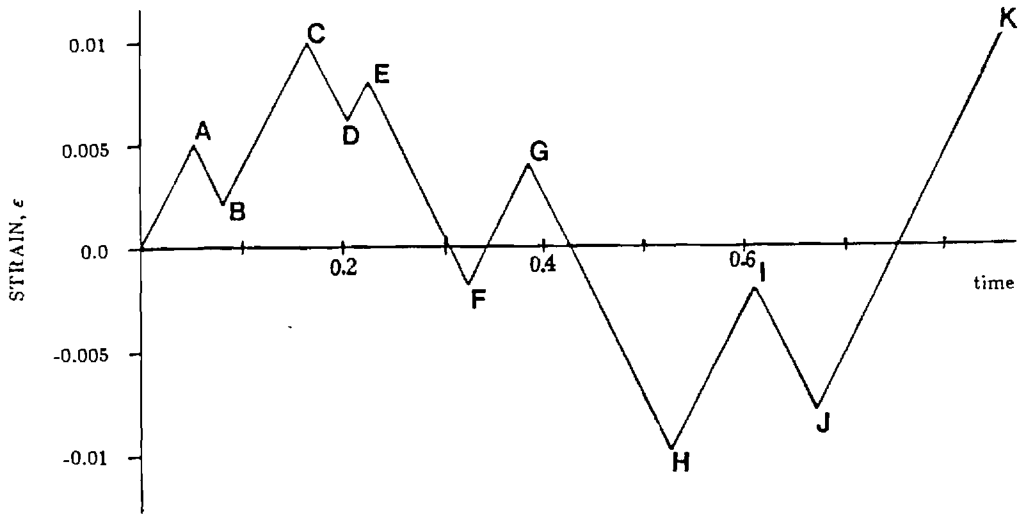

NODIM. STRESS, σ/σ₀
0.0
0.5
1.0
1.5
2.0
2.5
0.00
0.02
0.04
0.06
0.08
STRAIN, ε
STRAIN RATE
□ = 0.1/sec
○ = 1.0/sec
△ = 5.0/sec



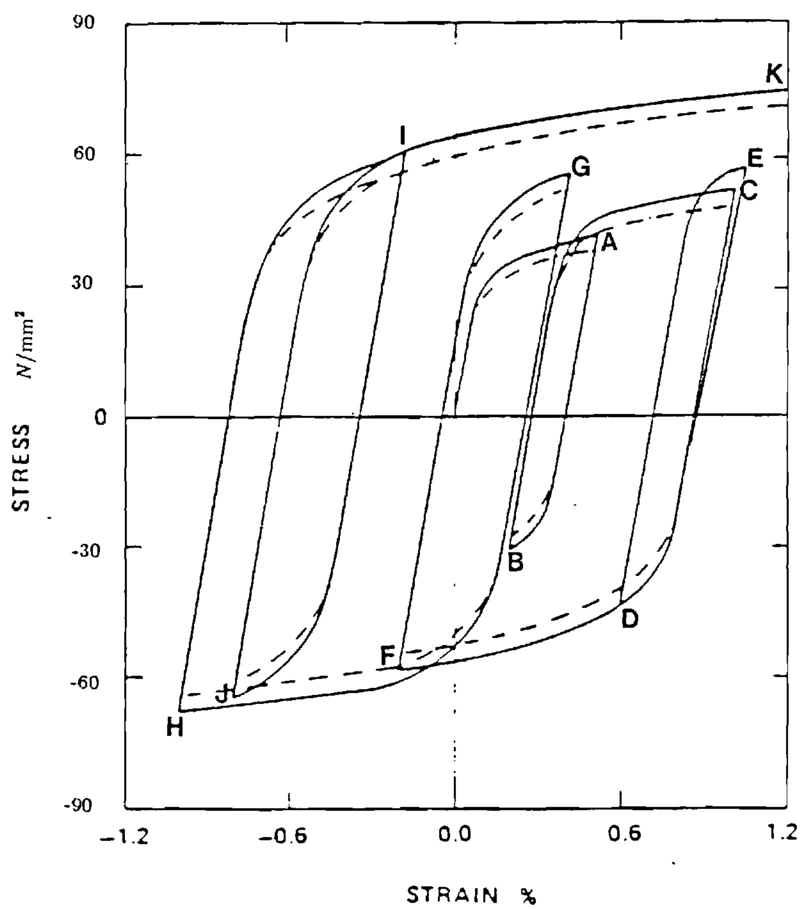
1.5
1.0
0.5
0.0
NODIM. STRESS, σ/σ_0
0.00
0.02
0.04
0.06
0.08
STRAIN, ϵ
RATE HISTORY (1/sec)
1.0, −1.0, 0.1
0.1, − 0.1, 1.0



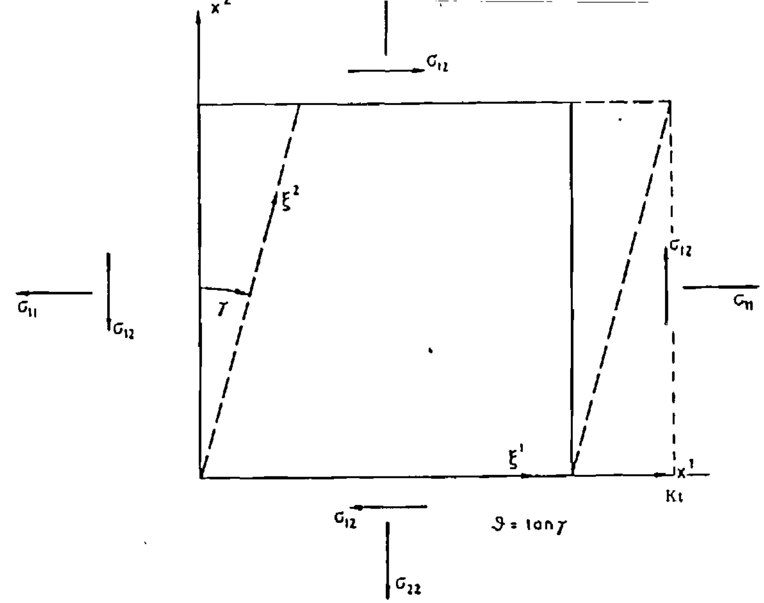
1.5
1.0
0.5
0.0
NODIM. STRESS, σ/σ_0
0.00
0.02
0.04
0.06
0.08
STRAIN, ε
RATE HISTORY (1/sec)
1.0, 0.1
0.1, 1.0



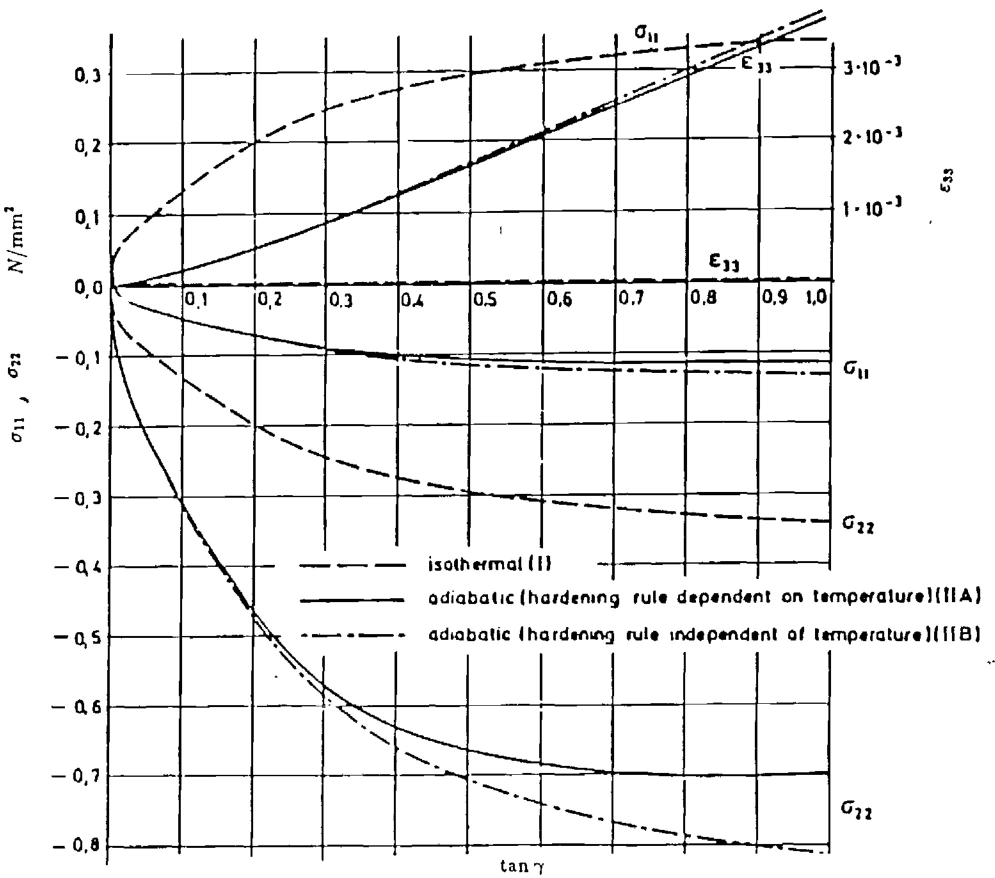
0.01
0.005
0.0
-0.005
-0.01
STRAIN, ε
0.2
0.4
0.6
time
A
B
C
D
E
F
G
H
I
J
K



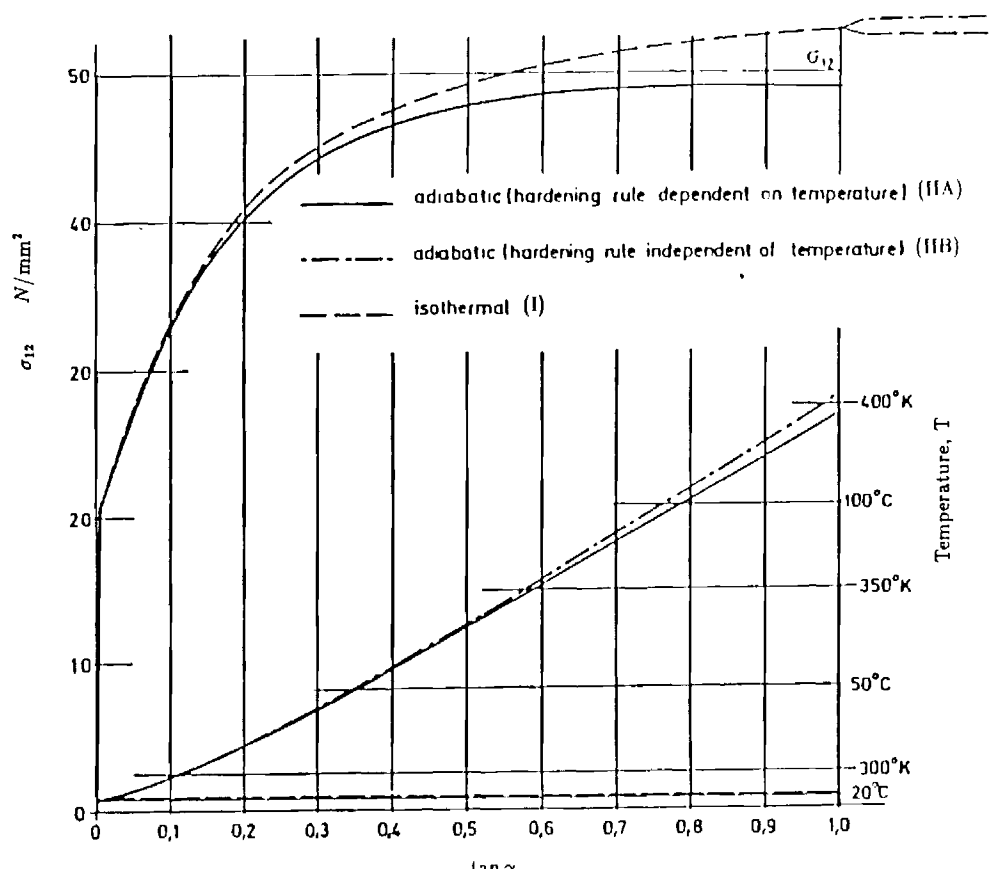
90
60
30
0
-30
-60
-90
STRESS N/mm²
−1.2
−0.6
0.0
0.6
1.2
STRAIN %
A
B
C
D
E
F
G
H
I
J
K



x^2
σ_{12}
ξ^2
γ
σ_{11}
σ_{12}
σ_{12}
σ_{11}
ξ^1
x^1
Kt
σ_{12}
$\vartheta = \tan\gamma$
σ_{22}



σ_{11} , σ_{22} N/mm²
ε_{33}
σ_{11}
σ_{22}
isothermal (I)
adiabatic (hardening rule dependent on temperature) (IIA)
adiabatic (hardening rule independent of temperature) (IIB)
tan γ



σ_{12} N/mm²
σ_{12}
adiabatic (hardening rule dependent on temperature) (IIA)
adiabatic (hardening rule independent of temperature) (IIB)
isothermal (I)
Temperature, T
400°K
100°C
350°K
50°C
300°K
20°C

NON-ISOTHERMAL ELASTOVISCOPLASTIC SNAP-THROUGH AND CREEP BUCKLING OF SHALLOW ARCHES

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Abstract

The problem of buckling of shallow arches under transient thermomechanical loads is investigated. The analysis is based on nonlinear geometric and constitutive relations, and is expressed in a rate form. The material constitutive equations are capable of reproducing all non-isothermal, elasto-viscoplastic characteristics. The solution scheme is capable of predicting response which includes pre and postbuckling with creep and plastic effects. The solution procedure is demonstrated through several examples which include both creep and snap-through behavior.

I. Introduction

Elastic snap-through of low arches under quasi-static loads has been the subject of several investigations. The significance of the problem, in so far as it illustrates certain important features in more complicated buckling problems of plates and shells, was pointed out by Marguerre [1], who constructed a simplified mechanical model to demonstrate these features. Timoshenko [2] obtained an approximate solution to the problem of a low arch under a uniformly distributed load. Biezeno [3] considered the problem of a low parabolic arch loaded laterally at the mid-point with a concentrated load. He also investigated snap-through buckling of a shallow spherical cap, pinned along its circular boundary, under the action of a concentrated load applied along the axis of rotational symmetry. He presented his approximate solutions by means of load-deflection curves and equations from which the critical load could be calculated.

In 1952, Fung and Kaplan [4] investigated the problem of low-pinned arches of various initial shapes and spatial distributions of the lateral load. Their results show that a very shallow arch snaps-through symmetrically, whereas a higher arch buckles asymmetrically. They also ran a limited number of experimental tests, and their experimental data are in good agreement with their theoretical results. About the same time, Hoff and Bruce [5], in investigating the possibility of snap-through buckling of a low pinned arch with a half-sine-wave initial shape under a half-sine-wave distributed dynamic load (suddenly applied with infinite duration), obtained results for the quasi-static load case which are identical to those obtained by Fung and Kaplan for the same problem.

In 1962, Gjelsvik and Bodner [6] obtained an approximate solution to the problem of a low circular arch with a concentrated load at the mid point of the arch and clamped boundary conditions. They also reported on experimental tests, and there is good agreement between their theoretical predictions and their experimental results. Schreyer and Masur [7] obtained an exact solution to their problem (and for the load case of uniform pressure), and they showed that for the concentrated load case,the arch snaps-through symmetrically regardless of the value of the rise parameter. Masur and Lo [8] presented a general discussion of the behavior of the shallow circular arch regarding buckling, post buckling and imperfection sensitivity. Snapping of low pinned arches resting on an elastic foundation has been investigated by Simitses [9]. This system exhibits all forms of experimentally observed buckling phenomena (smooth and violent) and of theoretically predicted responses (limit point, bifurcation with stable branching and bifurcation with unstable branching), and it is presented with sufficient detail in Ref. [10]. Experimental results have also been reported by Roorda [11].

The effects of inelastic material behavior found their way into the literature since the 1960's. Onat and Shu [12] considered the behavior to be one of rigid perfectly plastic. Franciosi, Augusti and Sparacio [13] discussed the collapse of arches under repeated loads with inelastic material behavior. Studies of inelastic snap through buckling of shallow arches were also reported by Lee and Murphy [14]. In addition Augusti [15] investigated plastic buckling of a model of a three hinged arch in 1968, and a more complete analysis of the same model was provided by Batterman [16] in 1971. Finally, the reader who is interested in the ultimate strength of parabolic steel arches with bracing system is referred to Komatsu [17], who considers inelastic in plane and out-of-plane instabilities and provides design formula for each case.

The elastic response of arches under sudden (dynamic) application of the external loads has been reported by Hoff and Bruce [5], Hsu [21], [22], and Lock [23]. For a more complete bibliography see Ref. 24.

Creep buckling of shallow arches has been investigated by Huang and Nachbar [18], Krajcinovic [19], and Botros and Bienek [20].

As far as the authors know no work has been reported on the non isothermal elastovisco plastic behavior of shallow arches. The purpose of this paper is to demonstrate the effect of highly nonlinear material behavior on the snap through and creep buckling of shallow arches.

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[2] Assistant Professor, Member AIAA

II. Elasto-Thermo-Viscoplastic Constitutive Relations

The prediction of buckling loads and postbuckling behavior of structural components, like shallow arches, made of a realistic metallic material and subjected to non-isothermal thermomechanical loads has increased in importance in recent years.

Under this kind of severe loading conditions, the structural behavior is highly nonlinear due to the combined action of geometrical and physical nonlinearities. On one side, finite deformation in a stressed structure introduces nonlinear geometric effects. On the other side, physical nonlinearities arise even in small strain regimes, whereby inelastic phenomena play a particularly important role. From a theoretical standpoint, nonlinear constitutive equations should be applied only in connection with nonlinear transformation measures (implying both deformation and rotations). However, in almost all of the works in this area [25], the two identified sources of nonlinearities are always separated. The separation yields, at one end of the spectrum, problems of large response, while at the other end, problems of viscous and/or non-isothermal behavior in the presence of small strain.

The classical theories, in which the material response is characterized as a combination of distinct elastic, thermal, time independent inelastic (plastic) and time dependent inelastic (creep) deformation components cannot explain some phenomena, that can be observed in complex thermo-mechanical loading histories. This is particularly true when high-temperature non-isothermal processes must be taken into account. There is a sizeable body of literature [25], [26] on phenomenological constitutive equations for the rate and temperature dependent plastic deformation behavior of metallic materials. However, almost all of these new "unified" theories are based on small strain theories and several suffer from some thermodynamic inconsistencies.

In a previous paper [27], the authors have presented a complete set of constitutive relations for nonisothermal, large strain, elasto-viscoplastic behavior of metals. It was shown there [27] that the metric tensor in the convected (material) coordinate system can be linearly decomposed into elastic and (visco) plastic parts. So a yield function was assumed, which is dependent on the rate of change of stress on the metric, on the temperature and on a set of internal variables. Moreover, a hypoelastic law was chosen to describe the thermo-elastic part of the deformation.

A time and temperature dependent viscoplasticity model was formulated in this convected material system to account for finite strains and rotations. The history and temperature dependence were incorporated through the introduction of internal variables. The choice of these variables, as well as their evolution, was motivated by thermodynamic considerations.

The nonisothermal elasto-viscoplastic deformation process was described completely by "thermodynamic state" equations. Most investigators [25], [26] (in the area of viscoplasticity) employ plastic strains as state variables. The authors' previous study [27] shows that, in general, use of plastic strains as state variables may lead to inconsistencies with regard to thermodynamic considerations. Furthermore, the approach and formulation employed in previous works leads to the condition that all the plastic work is completely dissipated. This, however, is in contradiction with experimental evidence, from which it emerges that part of the plastic work is used for producing residual stresses in the lattice, which, when phenomenologically considered, causes hardening. Both limitations were excluded from this [27] formulation. Accuracy of the formulation was checked on a wide range of examples [28].

The constitutive relation will be rephrased here in some different form. For brevity we compile only some notations and fundamental relations which are used in the formulation of the intended constitutive low. For details, see [27] and [28].

Concerning the formulation of constitutive laws it is advantageous to use a material (co-moving) coordinate system. The transformation from the underformed state (metric $\overset{\circ}{g}_{ik}$) to the deformed state can be represented by the tensor:

$$f^i_k = \overset{\circ}{g}{}^{ir}\, g_{rk} \quad or \quad (f^{-1})^i_k = g^{ir}\, \overset{\circ}{g}_{rk} \qquad (1)$$

The total deformation rate is defined by

$$d^i_k = \frac{1}{2} g^{ir}\dot{g}_{rk} = -\frac{1}{2} g_{kr}\dot{g}^{rk} = \frac{1}{2}(f^{-1})^i_r(\dot{f})^r_k = -\frac{1}{2}(\dot{f}^{-1})^i_r f^r_k \qquad (2)$$

here ($\dot{}$) denotes time material derivative. The expression

$$\overset{\nabla}{f}{}^i_k = (\dot{f})^i_k - d^i_r f^r_k - d^r_k f^i_r = sym\{(\dot{f})^i_k\} \qquad (3)$$

represents the symmetric part of $(\dot{f})^i_k$ or the covariant derivative with respect to time, also called the convected derivation, which is due to Zaremba and Jaumann.

The total deformation can be decomposed according to

$$f^i_k = g^{im}\overset{\bullet}{g}_{mr}\, g_{rk} = \overset{(1)}{f}{}^i_r\, \overset{(2)}{f}{}^r_k \qquad (4)$$

where the superscript ($\bullet$) relates to a fictitious configuration defined by a fictitious reversible process with frozen internal variables. The decomposition of Eq. (4) leads to an additive decomposition of the deformation rate

$$d^i_k = \overset{(r)}{d}{}^i_k + \overset{(i)}{d}{}^i_k \qquad (5)$$

$\overset{(r)}{d}{}^i_k$ is related to the reversible deformations, whereas $\overset{(i)}{d}{}^i_k$ denotes the remaining part of the deformation rate.

For the description of the stress state, we introduce the Kirchoff stress tensor s^i_k, which is connected with the real Cauchy stress tensor σ^i_k, by the relation:

$$s^i_k = \frac{\overset{\circ}{\rho}}{\rho}\sigma^i_k \qquad (6)$$

Assuming that the elastic behavior is not affected essentially by inelastic deformations, the following hypoelastic incremental law was chosen [27]

$$\overset{(r)}{d}{}^i_k = \frac{1}{2G}\overset{\nabla}{t}{}^i_k + \{\frac{1}{9K}\dot{s}^r_r + \alpha\dot{T}\}\delta^i_k \qquad (7)$$

with

t^i_k : weighted stress deviator
G : shear modulus
K : bulk modulus
α : coefficient of thermal expansion

The following constitutive relations were established [27] for the inelastic behavior.
yield condition:

$$F = (t^i_k - c\,\bar{\rho}\, g_i\beta^i_k)(t^k_i - c\,\rho\, g_i\beta^k_i) - k^2(A, T) = f^2 - k^2 = 0 \qquad (8)$$

accompanying equilibrium state:

$$\overline{F} = (\bar{t}^i_k - c\,\mathring{\rho}\,g\beta^i_k)(\bar{t}^k_i - c\,\mathring{\rho}\,g\beta^k_i) - k^2(A,T) = \bar{f}^2 - k^2 = 0 \quad (9)$$

evolution law for inelastic deformations:

$$\overset{(i)}{d^i_k} = 2\dot{\lambda}(\bar{t}^i_k - c\,\mathring{\rho}\,g\beta^i_k) \qquad (10)$$

with

$$\dot{\lambda} = \frac{1}{4\eta}\left(\sqrt{\frac{(t^i_k - c\,\mathring{\rho}\,g\beta^i_k)(t^k_i - c\,\mathring{\rho}\,g\beta^k_i)}{k^2}} - 1\right) \qquad (11)$$

and

$$\bar{t}^i_k = \frac{1}{1+4\eta\dot{\lambda}}(t^i_k - c\,\mathring{\rho}\,g\beta^i_k) + c\,\mathring{\rho}\,g\beta^i_k \qquad (12)$$

evolution laws for the internal variables:

$$\dot{A} = \frac{1}{\mathring{\rho}}\bar{t}^i_k\,\overset{(i)}{d^k_i} \qquad (13)$$

$$\overset{\nabla}{\beta^i_k} = \xi\,\overset{(i)}{d^i_k} \qquad (14)$$

if

$$F = 0 \quad and \quad \frac{\partial F}{\partial s^i_k}\overset{\nabla}{s^i_k} + \frac{\partial F}{\partial T}\dot{T} > 0 \qquad (15)$$

then

$$d^i_k = \overset{(r)}{d^i_k} \qquad (16)$$

$$\overset{(i)}{d^i_k} = 0 \quad and \quad \overset{\nabla}{d^i_k} = 2\ddot{\lambda}(t^i_k - c\,\mathring{\rho}\,g\beta^i_k) \qquad (17)$$

with

$$\ddot{\lambda} = \frac{1}{8\eta k^2}\left\{2(t^i_k - c\,\mathring{\rho}\,g\beta^i_k)\overset{\nabla}{t^k_i} - \frac{\partial k^2}{\partial T}\dot{T}\right\} \qquad (18)$$

if

$$F = 0 \quad and \quad \frac{\partial F}{\partial s^i_k}\overset{\nabla}{s^i_k} + \frac{\partial F}{\partial T}\dot{T} \leq 0 \qquad (19)$$

or

$$F < 0 \qquad (20)$$

then

$$\begin{aligned} d^i_k &= \overset{(r)}{d^i_k} \\ \dot{A} &= 0 \\ \overset{\nabla}{\beta^i_k} &= 0 \end{aligned} \qquad (21)$$

Within the developed frame the elasto-viscoplastic behavior is governed by the scalar material functions $c(s^i_k, T, A, \beta^i_k)$, $k^2(A,T)$, $g(s^i_k, A, T, \beta^i_k)$, $\xi(A, T, \beta^i_k)$, and $\eta(A, T, \beta^i_k)$. These material functions can be determined from a series of monotonic and cyclic processes with proportional and nonproportional paths at different temperature levels [28].

III. General Formulation and Solution Schemes

The rate form of the constitutive equations suggests that a rate approach be taken toward the entire problem so that flow is viewed as history dependent process rather than an event. For this purpose, a complete true ab-initio rate theory of kinematics and kinetics for continuum and curved thin structures, without any restriction on the magnitude of the transformation was presented in Ref. 28. It is implemented with respect to a body- fixed system of convected coordinates, and it is valid for finite strains and finite rotations. The time dependence and large strain behavior are incorporated through the introduction of the time rates of change of the metric and of the curvature.

The constitutive law may be applied to the conservation of momentum via an appropriate variational principle. The principle of virtual power (or of virtual velocities) reads

$$\int_V \sigma^{ij}\delta v_{j,i}\,dV - \int_V \rho f^j \delta v_j\,dV - \int_A \nu T^j \delta v_j\,dA = 0 \qquad (22)$$

where δv_j are the virtual velocities, f^j the body forces per unit mass and νT^j the surface tractions. Total differentiation of Eq. (22) yields,

$$\begin{aligned} &\int_V \left(\frac{d\sigma^{ij}}{dt} + \sigma^{ij}d^k_k - v^i_{,k}\sigma^{kj}\right)\delta v_{j,i}\,dV - \int_V \rho\frac{df^j}{dt}\delta v_j\,dV \\ &- \int_A \nu\frac{dT^j}{dt}\delta v_j\,dA + \int_V \sigma^{ij}\left(\frac{d\delta v_j}{dt}\right)_{,i}dV - \int_V \rho f^j\frac{d\delta v_j}{dt}\,dV \\ &- \int_A \nu T^j\frac{d\delta v_j}{dt}\,dA = 0 \end{aligned} \qquad (23)$$

At any instant, Eq. (23) must be satisfied. The virtual velocity and its time derivative are then independent. Moreover, the last three terms of Eq. (23) are equivalent to Eq. (22). Hence, the principle of the rate of virtual power may be obtained in its concise form. For further classifications, the total derivative of the stress components will be represented by the Jauman derivative, and the following integrals are defined by

$$I_s = \int_V \overset{\nabla}{\sigma^{ij}}\,\delta v_{j,i}\,dV \qquad (24)$$

$$I_d = \int_V (\sigma^{ij}d^k_k - \sigma^{kj}d^i_k)\delta v_{j,i}\,dV \qquad (25)$$

$$I_r = \int_V \omega^i_k\sigma^{kj}\delta v_{j,i}\,dV \qquad (26)$$

Then, substitution in Eq. (23) yields the final form of the principle of the rate of virtual power,

$$I = I_s + I_d - I_r = \int_V \rho\frac{df^j}{dt}\delta v_j\,dV + \int_A \nu\frac{dT^j}{dt}\delta v_j\,dA \qquad (27)$$

The quasi-linear nature of the principle of the rate of virtual power suggests the adoption of an incremental approach to numerical integration with respect to time. The availability of the field formulation provides assurance of the completeness of the incremental equations and allows the use of any convenient procedure for spatial integration over the domain V. In the present instance the choice has been made in favor of a simple first order expansion in time for the construction of incremental solutions from the results of finite element spatial integration of the governing equations.

The procedure employed permits the rates of the field formulation to be interpreted as increments in the numerical solution. This is particularly convenient for the construction of incremental boundary condition histories.

The finite element method for spatial discretization has been well documented (see, e.g. Zienkiewicz [29] or Oden [30]) and will not be detailed here. Restricting attention to a single finite element B_m, the velocity field is approximated by $\tilde{v}_i(x^j)$,

$$v_i(x^j) \approx \tilde{v}_i(x^j) = (\Gamma^{\alpha\beta})^T v^\alpha_i(x^j) V^\beta \qquad \alpha, \beta = 1, \ldots, N \qquad (28)$$

In Eq. (28), V^{β} are generalized nodal velocities, $\Gamma^{\alpha\beta}$ is dependent upon the present nodal coordinates, $v_i^{\alpha}(x^j)$ is a vector of prescribed functions of (x^j), and N is the number of degrees of freedom associated with the element. The matrix $\Gamma^{\alpha\beta}$ is defined by requiring that evaluation of $\bar{v}_i$ at nodal positions yields V^{β}.

The algebraic counterpart of Eq. (27) after the finite element discretization is the well-known stiffness expression

$$[K]\{V\} = \{\dot{P}\} - \{\dot{F}\} \tag{29}$$

with the tangent stiffness matrix $[K]$, the vector of the incremental velocities $\{V\}$, and the vector out-of-balance force rates, external force rates $\{\dot{P}\}$ minus internal force rates $\{\dot{F}\}$. The homogenoüs case of Eq. (29) indicates either the non-uniqueness of the equilibrium path at a stable or unstable bifurcation point or the unique but unstable situation at a limit point. Hence this criterion may be evaluated by a determinant check or supplementary eigenvalue analysis for the load parameter parallel to the loading process.

Even under the condition of static external loads and slowly growing creep effects, the presence of snap-through buckling makes the inertia effects significant. In dynamic analyses, the applied body forces include inertia forces. Assuming that the mass of the body considered is preserved, the mass matrix can be evaluated prior to the time integration using the initial configuration.

Finite element solution of any boundary-value problem involves the solution of the equilibrium equation (global) together with the constitutive equation (local). Both equations are solved simultaneously in a step by step manner. The incremental form of the global and local equations can be achieved by taking the integration over the incremental time step $\Delta t = t_{j+1} - t_j$. The rectangular rule has been applied to execute the resulting time integration.

Clearly, the numerical solution involves iteration. A simplified version [31] of Riks Wempner constant-arc-length method has been utilized. This iteration procedure which is a generalization of the displacement control method also allows to trace the nonlinear response beyond bifurcation points. In contrast tional Newton-Raphson techniques, the iteration of the method takes place in the velocity and load rate space. The load step of the first solution in each increment is limited by controlling the length ds of the tangent. Either the length is kept constant in each step or it is adapted to the characteristics of the solution. In each step the triangular-sized stiffness matrix has to be checked for negative diagonal terms, indicating that a critical point is reached.

IV. Shallow Circular Clamped Arch

The theory and computational procedures described in the preceding sections have been applied to the creep buckling analysis of a shallow circular clamped arch. The problem of the clamped arch beside being of some practical interest contains a number of similarities to that of the uniformly loaded spherical cap. The trend of results of the arch problem serves as a good indicator to the behavior of the latter.

The shallow circular clamped arch subjected to a single central concentrated load, as shown in Fig. 1, is analyzed. The material chosen for the numerical experimentation is the carbon steel C-45 (DIN 1720) with $E = 10^7$ psi, $\nu = 0.3$ and $\sigma_y = 2.7 \times 10^4$ psi at room temperature. The viscoplastic properties (the scalar material functions) were obtained in Ref. 28

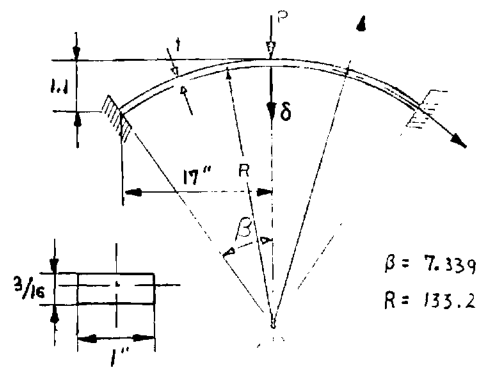
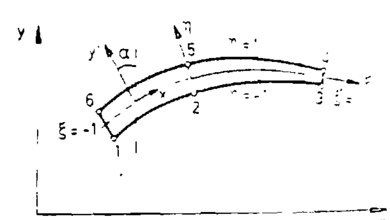


Fig. 1 Clamped Circular Arch

The analysis is performed with the aid of 24 paralinear isoparametric elements, Fig. 2. The paralinear isoparametric element is intended for the analysis of oriented structures where the geometry is such that the thickness is small compared to other dimensions. The characteristics of the element are defined by the geometry and interpolation functions, which are linear in the thickness direction and parabolic in the longitudinal direction (see Fig. 2). Consequently, the element allows for shear strain energy since normals to a mid surface before deformation remain straight, but not necessarily normal to the mid surface after deformation.



$\xi_o = \xi \cdot \xi_i$
$\eta_o = \eta\ \eta_i$

Node	N_i
Corner $\xi_i = \pm 1\ \ \eta_i = \pm 1$	$\frac{1}{4}(\xi_o + \xi^2)(1 + \eta_o)$
Midside $\xi_i = 0,\ \eta_i = \pm 1$	$\frac{1}{2}(1 - \xi^2)(1 + \eta_o)$

Fig. 2 Paralinear Isoparametric Element

The elastic behavior, corresponding to both axisymmetric and asymetric response, is shown on Fig 3. These curves are in complete agreement with those produced by Gjelsvik and Bodner [6], only because the young's modulus and Poisson's ratio values used are virtually the same (carbon steel C-45 here, and

2024-T4 aluminum alloy in Ref. 6). Note that these elastic response curves are hypothetical for our material but true for the 2024-T4 alloy. The true behavior for our material is elastoviscoplastic and it is labeled as such on Fig. 3. Note that this curve represents quasi-static (steady state) elastovisco- plastic response, as described by the chosen constitutive law. According to this, snapping occurs at a load of 26.20 lbs, primarily because of the low yield strength. Then, the post-limit point behavior seems to be primarily driven by viscoplastic behavior.

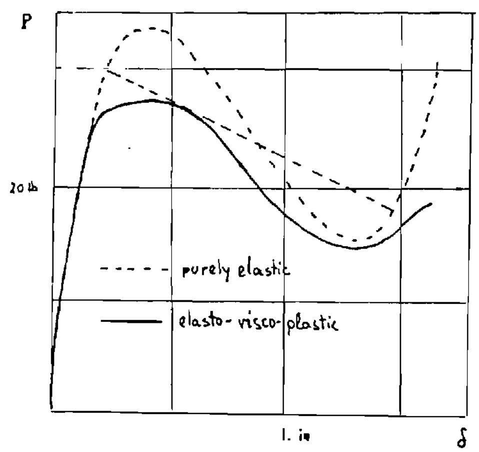


Fig. 3 The Arch Response

It is expected here that if loads up to approximately 14 lbs are reached quasi-statically and left applied for a long time the primary response will be creep and the critical time to creep will be extremely large. On the other hand, for loads between 14 lbs and 26.2 lbs (especially for the higher range) the behavior will be a combination of creep and snap-through buckling. This is best demonstrated by the curve on Fig. 4. The applied load is reached quasi-statically in 13 minutes and then it is kept constant. The curve of Fig. 4 depicts the behavior of the arch in terms of midpoint deflection versus time. Creep, initially, is very slow, then snap-through takes place in 32 minutes, curve BC, and then the creep behavior continues until a critical time to creep (creep buckling occurs) is reached after a total time of 97 minutes. Note that for this loading condition the critical time to creep is 97 minutes. Creep buckling and critical time to creep are based on the phenomenon that the deflection increased very rapidly. For loads higher than 26.2 lbs, it is expected that snapping will occur early, during quasi- static loading and then the creep behavior will be similar to that shown on Fig 4, past point c.

The next example considers the influence of cyclic loading on the response. Figure 5 illustrates the load deflection behavior of the arch under a cyclically applied external load. The load is increased, quasi-statically, from zero to 26 lbs in 5 minutes, then it is held constant for 2.5 minutes, after that it is reduced to 20 lbs and held constant for 50 minutes, then raised to 25.5 lbs for 2.5 minutes and finally it is reduced back to 20 lbs and held constant.

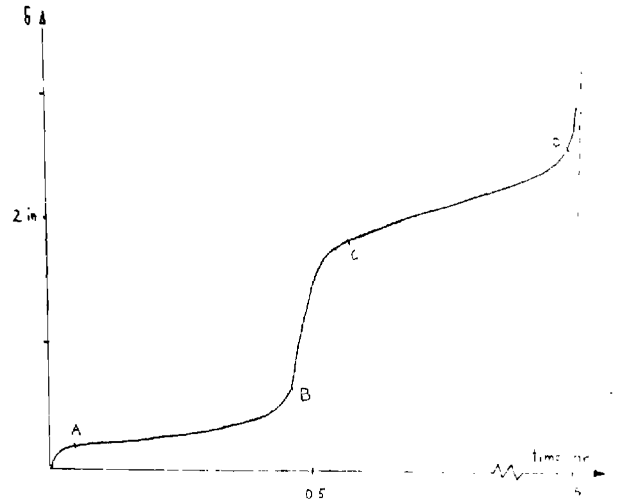


Fig. 4 Deflection Time History

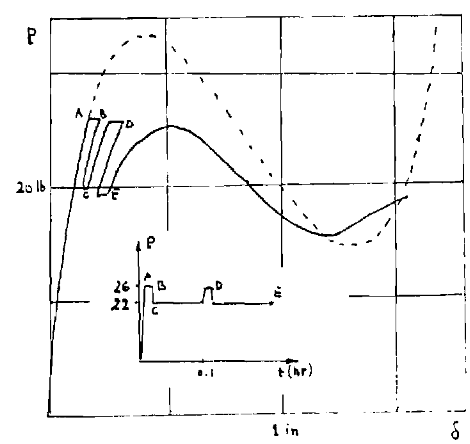


Fig. 5 Multi-Cycle Arch Response

The steady state response under this type of loading exhibits several limit points, which may imply that snapping is imminent shortly after the load reaches the value of 26 lbs (between points A and B on Fig 5). The dashed curve corresponds to the hypothetical elastic static response and it is only shown for comparison purposes.

The last example presented in Fig 6, consider the influence of temperature on the arch behavior. The loading history is the same on the one shown on Fig. 4. The curve corresponding to $T = 50^\circ F$ was discussed previously (Fig. 4), and it is used here as a basis for comparison. When the temperature is raised to $200^\circ F$ (after this, the loading is applied), the time to snap is reduced to 26 minutes, while the critical time to creep is not appreciable affected. On the other hand at the highest temperature $T = 1000^\circ F$, for which results are shown. The critical time to creep is reduced to 62 minutes, and the steady state response does not show a clear snap through behavior.

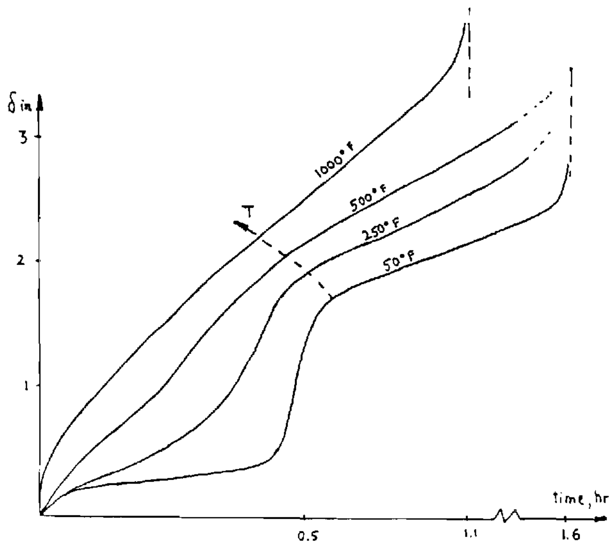


Fig. 6 The Influence of Temperature Raise

V. Discussion

As noted earlier, the main thrust of this work has been to demonstrate the effect of highly nonlinear material behavior on the snap–through and creep buckling of shallow arches. It is evident that in the presence of both elastic and viscoplastic deformation the process of buckling assumes an entirely new character. Buckling develops as a time–temperature dependent deformation process under constant or variable loads of magnitudes smaller than the elastic critical values. In arches under loads below the critical values the structure initially deform quasi-statically with the thermo-viscous terms manifesting themselves in the form of increasing displacement under, say a constant load. When the magnitude of the displacements reaches a certain threshold state, the arch snaps dynamically into the post-buckling configuration and then continues quasi-static deformation again.

The material constitutive relation has been proven to be capable of reproducing the main characteristics of viscoplastic deformations. The modified Riks/Wempner iteration scheme has been found to be a versatile technique in the pre-and post critical range.

The influence of thermo-mechanical coupling can become very large in stability problems. Such processes are always connected with a rapid growth of inhomogeneity of the state. Thorough investigation of such problems, however, must be performed with the necessary detail.

Acknowledgement

The research described above has been performed under NASA Grant No. 3-534. The financial support provided by NASA is gratefully acknowledged by the authors. The authors also wish to extend their thanks and appreciation to Dr. C.C. Chamis of the NASA-Lewis Research Center for his support and for the insights into the problem which he provided during our many technical discussions.

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APPENDIX B

Computer Codes

ABAQUS [B1],[B2]

ABAQUS is a modern general purpose linear and nonlinear static and dynamic three-dimensional finite element analysis program. The nonlinearities may be due to large displacement, large strains or any nonlinear material behavior.

ABAQUS offer a very large range of applications in linear and nonlinear analysis with relatively few modern very effective elements, a good library of material models and state-of-the-art numerical methods. Numerous user interfaces for specification of user selected parameters and subroutines make the program extremely flexible.

ADINA [B3],[B4],[B5]

ADINA is general purpose linear and nonlinear static and dynamic three-dimensional finite element analysis program. The code was developed on experience obtained from codes SAP-IV and NONSAP. Development started in 1974 and the code is continuously improved.

The nonlinearities may be due to large displacement, large strain, and nonlinear material behavior. The material behavior include thermo-elastic-plastic-creep model by so called effective stress-function algorithm. This algorithm is based on Von-Mises plasticity with isotropic and kinematic hardening plus potential creep theory. The algorithm calculates stress state for given strain and temperature.

ADINAT has been employed for the solution of heat transfer and analogous field problems.

ADINA and ADINAT offer a very large range of applications in linear and nonlinear analysis with relatively few effective elements, a good library of materials models and effective numerical methods. The programs contain state-of-the-art finite element procedures in the element kinematic formulation, the formulation and implementation of nonlinear material models and in the iteration procedures. The programs can be employed effectively in linear analysis, and then, with only a few input changes, in very complex nonlinear analysis.

ANSYS [B6],[B7]

ANSYS is a large-scale general purpose finite element code. The program capabilities include structural analyses (static and dynamic; elastic, plastic and creep; buckling; small and large deflection), and heat transfer analyses (study-state and transient; conduction; convection and radiation). Structural and heat transfer analyses may be made in one, two or three dimensions, including axisymmetric and plane problems. Coupled thermal-fluid flow capability, coupled thermal-elastic capability and fluid solid interaction capability are also available.

A single model may be used for heat transfer, static structural, and dynamic structural analyses. Temperature output from the heat transfer analysis is in the form required for input to the structural analyses.

All heat transfer element types may be deleted or replaced by geometrically equivalent structural elements for thermal-stress evaluation.

Dynamic analyses may be made on structures that have been pre-stressed under static loading conditions.

ASAS-NL [B8]

ASAS-NL is a proprietary code sponsored by a number of existing ASAS users and geared principally to the requirements of the nuclear industry.

ASAS-NL is design to solve nonlinear static (small strain) problems due to material or geometric nonlinearities quickly and efficiently. Particular attention has been directed at the facilities for controlling and adjusting the computational sequences. Over twenty-five elements, ten solutions and ten materials models are built in, thus providing wide ranging capabilities for plasticity, creep and swelling, large deflection and buckling.

ASKA [B9]

ASKA is a comprehensive software system mainly for linear static and dynamic analysis. However, some selected nonlinear applications are also covered, mainly elasto-plastic analysis and bifurcation buckling analysis. ASKA has a very powerful multilevel substructure analysis capability based upon a central data base concept. It is-capable of solving very large problems even on minicomputers.

DIAL [B10]

Dial is a general purpose finite element program with the ability to perform static and transient nonlinear analysis in addition to linear stress analysis, vibration and buckling.

Dial is a modern processor-oriented code with almost complete, up-to-date libraries of two and three-dimensional elements.

The system consists of several independent processors which communicate via a generalized data base and file management system.

Due to the modular data base concept, user control is very versatile and economical.

Nonlinear static solution techniques include many variations of Newton-type methods with automatic load step calculation. Nonlinear transient techniques include several implicit schemes using Newton methods and an explicit scheme.

JAC [B11]

JAC is a finite element computer program for solving the large deformation, temperature dependent static problems for two-dimensional bodies. Either plane strain of axisymmetric geometry may be used. Material properties include isothermal and temperature dependent elastic-plastic, secondary creep and soil large strain models. Sliding interfaces can also be modeled. Solutions are obtained with the use of a nonlinear conjugate gradient technique. To accelerate convergence of the solution process during a load step, a fraction of the displacement increment of the previous load step may be used as an initial incremental displacement to start the solution. The formulation uses isoparametric nine-node Lagrangian elements which are integrated with a nine point rule evaluated at the nodes of the grid. To also accelerate convergence, a solution of a load increment can first be obtained with the use of four-node elements by using only the corner nodes of the nine-node elements. The four-node results are then used as a starting displacement vector to obtain the nine node element solution. When geometric nonlinearities are present, it is sometimes advantageous to calculate a geometrically linear solution and then restart the solution to obtain a geometrically nonlinear solution. Therefore, when obtaining the final nine node element solution, first a linear solution and then a geometrically nonlinear solution can be obtained. The program is recommended for use on highly nonlinear analyses and not for linear solutions. It is much more efficient to solve linear problems another way.

LASTRAN 80 [B12]

LASTRAN 80 is a program system designed to provide a basis for nonlinear static and dynamic structural computations. Standard applications are the modified Newton-Raphson procedure, a third order Hermitian nonlinear dynamic algorithm and an explicit dynamic algorithm.

The finite element programs are easily attached to the system through a unified concept. An open concept allows user access in all levels of the program hierarchy.

MARC [B13]

MARC is designed for the linear and nonlinear analysis of structures in the static and dynamic regime. Its extensive element library makes it useful for elastic analysis and its broad coverage of structural mechanics makes it an invaluable nonlinear analysis tool. The following nonlinearities are handled by the program: elastic-plastic, large displacements, finite strain, creep, thermally dependent properties. An eigenvalue for buckling may be obtained after each load increment. Dynamic analysis can be carried out by the modal or the direct integration procedure. Anisotropic, elastomeric and incompressible material descriptions are available. Restart capability to restart analysis at any increment or time-step. A rigid-plastic flow capability and a fluid-solid modeling capability are available. The heat transfer uses element types of which a structural analog exists, making a decouple thermo-mechanical model using the same mesh possible. Extensive constraint and servo link capabilities are available. Numerous user interfaces for specification of user selected parameters make the program extremely flexible.

Marc code is used extensively by researchers working in the areas of "unified" viscoplastic material laws and high temperature applications. The current state-of-the-art of finite technology is adapted and incorporated into the program. New releases of the program are generated at the rate of about one per year.

MSC/NASTRAN

MSC/NASTRAN is an advanced version of the NASA-founded general purpose structural analysis program. In its initial design, the primary purpose of the NASTRAN program was the solution of linear elastic problem. However, substantial new nonlinear capabilities have been incorporated into MSC/NASTRAN. The newest system contains new, highly efficient solution code for both large displacement and nonlinear material analysis. Nonlinear static solution sequences provide for user-selected combinations of incremental, initial stress, Newton-Raphson and modified Newton-Raphson methods. The nonlinear transient procedure contains a limited capability for user-defined nonlinear load functions.

NEPSAP [B14],[B15]

NEPSAP is a general-purpose, three-dimensional, nonlinear finite element code capable of large displacements, thermo-elastic-plastic and creep analysis of arbitrary structures. The program can be used to model one-, two-and three-dimensional structural models composed of frame members, membranes, thick/thin plates and shells, isoparametric solids, or any combination of these. Although the code is formulated to account for both geometric and material nonlinearities, linear problems may be analyzed with no loss of efficiency. There is no inherent program limitation on the size of the models analyzed using NEPSAP. Analyses may be restarted at any load or time step. Finally, enhancements may be introduced rather easily due to the modular structure of the code.

Several pre-and post-processor modules are available as part of the NEPSAP system to enable the user to interface easily with the code. The model may be generated using either a user-written FORTRAN driver or by executing a very flexible preprocessor code which accepts key-word and list directed free field data cards. A complete graphics package enables the user to debug and verify the model rapidly as well as to display the analysis results. Utility modules are also available to interrogate and/or modify the contents of the random access data base files and for selective review of output data.

A variety of incremental and iterative strategies may be selected by the user for both nonlinear static and dynamic problems, with the default method being the incremental method using a built-in load correction feature. For transient dynamics. implicit time integrators used are (a) generalized Newmark, (b) Houbolt, (c) Park and (d) user-defined single or multi-step methods.

NISA II [B16]

NISA II is a general purpose linear and nonlinear, finite element analysis program for solving structural, heat transfer and fluid flow problems.

The nonlinearities may be due to large displacement or rotations, large strain and nonlinear material behavior. Material models include: Von-Mises, Tresca, Drucker-Prager and Mohr-Coulomb yield theories; elastic-perfectly plastic; Elastic-plastic with isotropic, or Kinematic hardening; Temperature dependent inelastic properties; and nonlinear stress strain data as piecewise linear or as Ramberg-Osgood curve. The program performs linear and nonlinear steady state and transient heat transfer analysis. The loading and boundary conditions may be time/temperature dependent.

The dynamic analysis includes: natural frequencies, modal shapes and modal stress analysis; transient dynamics; Shock spectrum, and Random vibrations.

SAMCEF [B17]

SAMCEF is a general purpose linear and nonlinear, static and dynamic, three-dimensional finite element analysis code. The program can perform a static analysis (linear and nonlinear) including the thermal effects, a dynamic analysis (linear and nonlinear) with computation of eigenvalues, eigenmodes and dynamic response, a stability analysis including buckling and postbuckling and a weight optimization analysis of structures subjected to several static and dynamic constraints. The finite element library is very vast. In addition to the general elements (truss, beam, plate, shell), displacement and stress type elements allow a dual analysis of various problems. Hybrid and mixed shell elements are also available. A few scalar field elements provide a solution to some fluid mechanics problems. In most of the elements the material can be anisotropic. The thicknesses, cross-sections, moments of inertia may vary linearly inside the element. Shell elements can be made of sandwich material. The nonlinearities may be due to large displacements, large strains and nonlinear material behavior. Any analysis may be performed step by step: data preparation, element generation, structural assembly, and solution.

SESAM-69 [B18],[B19]

SESAM-69 is a general purpose program system for linear and nonlinear analysis. Both for static and dynamic analyses the solution algorithm is based on the multilevel superelement-technique. Considerable advantages are obtained with this technique both for linear and nonlinear problems.

The total system is split into program modules of pipes, beams, membranes, shells and solids which may be executed autonomously or in combination with the superelement program. Nonlinearities include elasto-plasticity of three-dimensional membrane and solid structures, and combined large displacements and elasto-plasticity for shells. The program has complete saving-, restart-facilities. Vibration and dynamic analysis includes: computation of eigenvalues and eigenvectors, modal analysis and stepwise integration. Stability analysis includes: nonlinear collapse, and postbuckling analysis of shells.

STAGSC-1 [B20]

STAGSC-1 is a computer for structural analysis of shell type structures. It contains options for static stress analysis, bifurcation buckling, vibrations and transient response. Geometric and material nonlinearities may be included. While primarily intended for shell analysis, STAGSC-1 includes spring and beam elements. A number of substructures can be defined separately. Input is particularly simple when these belong to a set of standard geometries. Shell wall may consist of orthotopic layers with different orientation. discrete stiffeners may be included. Thermal as well as mechanical loading may be considered. The latter case includes options to define forces or displacements. Initial shape imperfections and cutouts may be defined.

For linear equations the skyline method (with Chlesky decomposition) is used. Eigenvalue problems are solved through the generation of invariant subspaces by simultaneous inverse power iteration. Nonlinear equations are solved by use of the modified Newton method with some automatic features for control of step size and relaxation factors. Tim integration is done by explicit (central differences) or implicit (trapezoidal and stiffly stable) methods.

STAGSC-1 does not include creep or visco-elasticity. It does not include three-dimensional elements. Boundary conditions are linear. Bifurcation buckling analysis does not include plasticity. Vibration analysis is linear. The nonlinear analysis is limited to "moderate rotation".

STRAW [B21]

STRAW was developed initially for determining the short duration transient, nonlinear response of core subassemblies for safety analysis in the Liquid Metal Fast Breeder Reactor Program. The program has since been developed along a more general purpose nature with recent emphasis on inclusion of quasi-Eulerian-fluid elements and linear and nonlinear thermal stress capability in the implicit time integration version. Problems are limited to two-dimensional or three-dimensional axisymmetric geometry. The material models are elastic and elastic-plastic with isotropic hardening and temperature varying properties.

TEPSA [B22]

TEPSA code is constructed on the basis of incremental variable stiffness finite element thermo-elasto-plasticity theory. Triangular and/or quadrilateral plates and torus elements are used for two-dimensional planar and three-dimensional axisymmetric structures. Both thermal and mechanical analyses are coupled; i.e. the thermal analysis is performed on the structural geometries at the immediate last load step. Code accepts temperature and strain-rate dependent material properties. Isotropic hardening rule is used for monotonic load increments and kinematic hardening rule for cyclic thermomechanical loadings. Special gap and crack elements are available for thermomechnical contacting surfaces and allow crack growth through elements. Code is constructed on the "modular" basis. Modular for special purposes such as finite strain plasticity, Fourier series coupling and thermomechnical creep can be readily adapted to the main program. The code can also handle solids involving phase change by special time-difference algorithm.

Incremental variable stiffness method is used for the nonlinear thermo-elastic-plastic analysis. Contact analysis is handled by the artificial noncompressive fluid concept. Successive reduction of stiffness is used for the crack propagation through elements.

WECAN [B23]

WECAN is a general purpose linear and nonlinear static and dynamic three-dimensional finite element analysis program. WECAN can solve static, modal, seismic response spectrum, harmonic response, linear and nonlinear dynamic transient, linear buckling, and heat transfer and analogous field problems. Isotropic, orthotopic, and anisotropic materials are permitted. Material properties are defined as a fifth order polynomial of temperature. WECAN can use the initial stress matrix to calculate static, modal or linear buckling problems. Substructures are linear but can be combined with nonlinear elements in the solution phase. Multilevel substructures are permitted. Substructures may be rotated, reflected or scaled. WECAN may be restarted at preselected time steps.

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APPENDIX C

Statement of Work

TECHNOLOGY ASSESSMENT

Structural Analysis of Hot, Highly-Loaded Structures

1.0 BACKGROUND

Design of future hypersonic aerospace vehicles will require reliable prediction of the combined effects of elevated temperature and large mechanical loads. These severe environments, resulting during vehicle operational mission profiles, can change dynamic and static characteristics of the structure. Weight-critical vehicle designs will be lightweight and flexible, and, therefore, highly susceptible to such changes in characteristics. Small displacement/small angle theory is inadequate for analysis of these structures since large displacements are expected to result from the high load conditions. Use of exotic, heat-resistent, nonlinear materials will further complicate structural analysis procedures.

Structural analysis techniques have been developed to account for changes in stiffness due to geometric and material nonlinear behavior during static load application and to perform linear dynamic analyses at this prestressed equilibrium state. These techniques are often ill-defined, inefficient, and costly. More efficient methods may be available, particularly in industries other than aerospace. The intent of this contract is to provide an independent assessment of existing state-of-the art technology for static and dynamic analyses of hot, highly-loaded structures.

2.0 STATEMENT OF WORK

The contractor shall evaluate current technology for the structural analysis, both static and dynamic, of hot, highly-loaded structures. This evaluation shall provide an independent assessment of the relevancy, validity, verification methodology, and shortcomings of current and envisioned analytic techniques. Topics shall include, but not be limited to:

(a) Computer code availability and suitability.

(b) Capability of these codes to handle the severe thermal and mechanical loading conditions for hypersonic vehicles. (McDonnell Douglas will provide information on these loading conditions.)

(c) Definition of nonlinearity and what constitutes large displacements/rotations.

(d) Susceptibility of structural types/configurations to nonlinear response.

(e) Validity of small strain/large displacement techniques.

(f) Assessment of various substructuring techniques.

(g) Relevant high temperature/high load test data availability for use in analysis validation.

(h) Envisioned trends in analytic/experimental research and development.

(i) Recommendations for future work.

The contractor shall submit the final report during the final coordination meeting at contractor facilities (see Paragraph 3.0). A draft of this report shall be received by the McDonnell Douglas contact at least one week prior to this meeting.

3.0 SCHEDULE AND DELIVERABLES

The anticipated period of performance of this contract will be from 15 October 1987 to 15 January 1988. An initial coordination meeting will be held at McDonnell Douglas, St. Louis, MO, on a mutually acceptable date prior to 10 November 1987. A final coordination meeting will be held at contractor facilities, on a mutually acceptable date prior to 15 January 1988. At this time, the contractor will present an oral summary of their findings. Additionally, the contractor shall provide five copies of a report detailing the results of this technology assessment. This report shall contain written descriptions of the study results and detailed listings of references and individuals working in the field of structural analysis of hot, highly-loaded structures.

4.0 CONTACT:

The McDonnell Douglas point of contact for this contract will be Paul W. Heaton at (314) 233-8369. Reports will be forwarded to the following address:

McDonnell Douglas Astronautics Company
P.O. Box 516
St. Louis, Missouri 63166

Attn: Paul W. Heaton
Technical Specialist
Dept. E466/HQ/4S/F-17